PEPPERMINT

薄荷

POH HOH

C.F. RUSSELL

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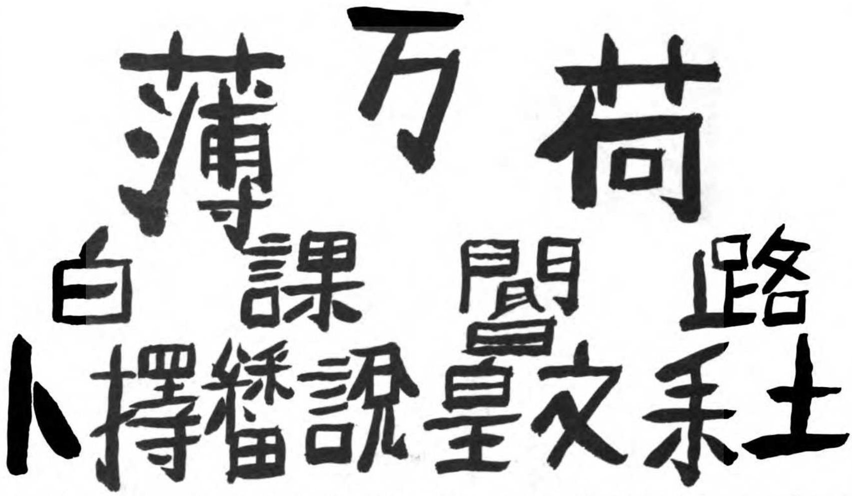
P E P P E R M I N T (P o h H o h)



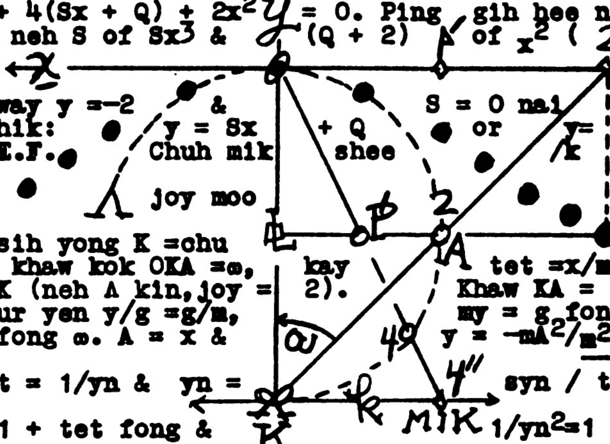
An Abso-Ming-Wen-lutely Absey-Booke		(9-30-54)	A156451
Book of Radicals	(Shu Pu)	(10-7-54)	A156452
Rime Store	(Yon Fer)	(11-12-54)	A161354
Rough Cloth	(Zao Jing)	(11-24-54)	A162915
Silver Cradle	(Yut Wo)	(12-21-54)	A165835
Dragon Pearls	(Maw Ni)	(1-17-55)	A169682
Gentle Rain	(Sun Zyu)	(2-1-55)	A171562
Wild Rice	(Yieh Mee)	(2-21-55)	A177026
Jade Mirror	(Yok Kyn)	(3-22-55)	A182077
Hill Road	(Sang Lo)	(4-16-55)	A182507
Spring Palace	(Jun Koong)	(5-14-55)	A186423
Water Chestnuts	(Chek Sak)	(6-11-55)	A190535
Open Door	(Tek Men)	(7-26-55)	A195705
Gnaw Through	(Zhi Gah)	(8-30-55)	A200071
Horseback	(Mar Siang)	(10-11-55)	A205249
Mud Hut	(Mao Sha)	(11-15-55)	A209990
North Well	(Bo Dzing)	(1-3-56)	A216860
Millet Valley	(Sah Kuk)	(2-14-56)	A223757
Copper Kettle	(Toong Du)	(3-20-56)	A228605
Peppermint	(Poh Hoh)	-56)	A

(0) SEEN. Dai ye sup Siao Shu Jih mun PAO. Fon cheek jak Loo, P O H H O; Chur, PAK FO HUEN LO; Liung, POOK JAK FAN SHUO HUANG WEN LOY TOE. Tzu wen tung peen (2).

(1) MIK SHIK. Dang joy lwen shik hook deem lo gih lwan mun Moo. Kon SANG LO (27) & gai kay mik; kon TOONG DU (43).

Yong tsen yuan A kay ping $\equiv x^2 + y^2 + 2y = 0$, chu 0, mwa of maw x & y = OK, kin tung chung L (0,−1) pan kin = r = 1. Wha jak / on K, sup lun A = A & sup x = A'; shik joy A = 2, A' = 2'. Tsung on 2', //y, sup heng on A, // x, = #P on Moo, M. Hur M ping $\equiv 2x^2 + x^2y + 4y = 0$. Wan gai kay mik? DAH. Khaw maw $\equiv y = Sx + Q$ (S = sia & Q = tsung y sup) Kon TEK MEN (30 Hao). Yong (Sx + Q) for y in siang M ping & yoo:

$x^2(Sx + Q) + 4(Sx + Q) + 2x^2 = 0$. Ping gih hee n, n−1, tzu x^2 & x^3 neh S of Sx^3 & (Q + 2) of x^2 (Q+2) tung for dah, neh S = 0 & Q + 2 = 0, way y = −2 & siang maw ping, shik: y = Sx + Q shee mik; Q.E.F. S = 0 nai or y = −2, Chuh mik shee joy.



Way gai AK M joy moo ping (0,0). Tsih yong K = chu hur khaw kok OKA = ω, whaw m = OK (neh A kin, joy = 2). kay tet = x/m. Khaw KA = g = m · yn ω. Hur yen y/g = g/m, my = g^2 fong & y = m · yn fong ω. A = x & $y = \overline{-mA^2/m^2 + A^2}$.

Nim fat: kat = 1/yn & yn = syn / tet; hur kat fong = 1 + tet fong & $1/yn^2 = 1 + tet^2$ & $yn^2 + yn^2 \cdot tet^2 = 1$. Hur $y/m + y/m\,(x^2/m^2) = 1$ & $ym^2 + yx^2 = m^3$ & $y(m^2 + x^2) = m^3$ & hur $y(x^2 + 4r^2) = 8r^3$, ah ping gih moo chu K (as 0,0). Ki, jeen chu yop 0, whaw x' = x & y' = y + 2 & yoo sin moo ping yong siang neh:

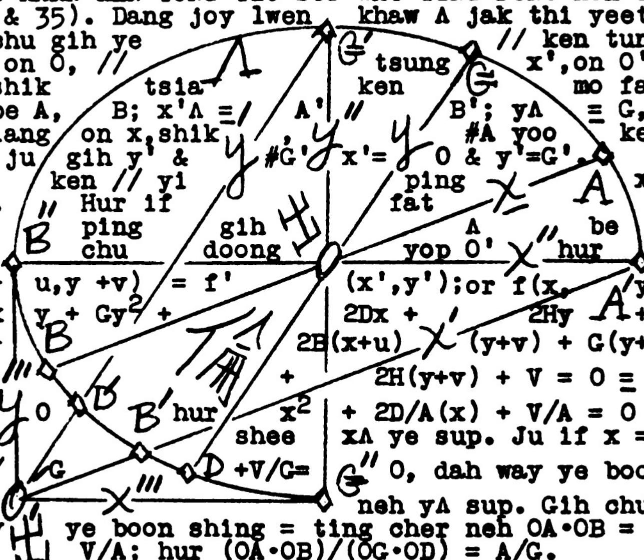
$(y + 2)(x^2 + 4) = 8$, or $x^2y + 2x^2 + 4y = 0$ meh. Kyn fat gih yuan A joy ($x' = x/e$ ($e = x^2 + (1 + y)^2$) & $y' = \frac{(1+y-e)}{e}$. Hur $M' \equiv 2x^2/e^2 + x^2/e^2(1+y-e)/e) + 4(1+y-e)/e = 0$, or

$(1 + y - e)(x^2 + 4e^2) + 2ex^2 = 0$, or $x^2(e+1+y) + 4e^2(1+y-e) = 0$.

Kyn fat way P' ken $\equiv$ x = 2/e (1/2) & y = −1. Yong nai M':

yoo $1/4(e) + 4e^2(-e) = 0$, or $e = 16e^3 = 1/4$, or $16e^2 = 1$. Nim e tzu P yong P ken & tzu P' yong P' ken way e tzu P = 4 & kay kyn P' yoo e = 1/4. MOO $\equiv$ WITCH of MARIA AGNESI.

(2) PING HARN KEN TUNG TZU SUP WAY TING FUN. Kon YOK KYN (33 & 35). Dang joy lwen khaw Λ jak thi yeet & O, O' ye jak chu gih ye // ken tung, neh tsung x,on O, // tsung x',on O' & ju y // y', shik tsia ken mo fat chan. Let xΛ be A, B; x'Λ = A', B'; yΛ = G,D & y'Λ: G',D'. Liang on x, shik #A yoo ken x = A, y = 0 & ju gih y' & #G' x' = 0 & y' = G'. Ju chuh. Yen tzu ken // yi ping fat $x' = x + u$, $y' = y + v$. Hur if yeet hing ping gih Λ be :f(x,y) then if chu doong yop O' hur sin fat $\equiv f'(x + u, y + v) = f'(x', y')$; or f(x,y) shik: $Ax^2 + 2Bxy + Gy^2 + 2Dx + 2Hy + V = 0$; $A(x+u)^2 + 2B(x+u)(y+v) + G(y+v)^2 + 2D(x+u) + 2H(y+v) + V = 0 \equiv f'(x',y')$. Khaw y = 0 hur $x^2 + 2D/A(x) + V/A = 0$ & OA,OB ye boon xΛ ye sup. Ju if x = 0, hur $y^2 + 2Hy/G + V/G = 0$, dah way ye boon y = OG, y = OD, neh yΛ sup. Gih chuh ye pun ping, kay ye boon shing = ting cher neh OA·OB = V/G & OG·OD = V/A; hur (OA·OB)/(OG·OD) = A/G.



Ju gai way (O'A'·O'B')/(O'G'·O'D') = A/G. Fo fat. Thus, if O be a variable # in conic plane & AB, GD, chords, have fixed directions through O, then OA·OB/OG·OD is a constant ratio (viz. no matter where O be, as at O, O', etc.

Nim cho! Fat chu Isaac Newton: Enumeratio linearum tertii ordinis (Opticks, 1704). Lun shik yong ching ken tung O neh x"Λ = A',B"; y"Λ = G',G" & tung O': x'"Λ = G",G"; y'"Λ = B"B". Hur (OA'·OB")/(OG'·OG") = (OG" fong/OB" fong), shik = 16/9. Joy gih O(x,y) ken $\Lambda \equiv 9x^2 + 16y^2 = 144$ (OA' = 4, OG' = 3). Khaw O' ken, x = −4, y = −3, hur, in O', O ken $\equiv x' = 4, y' = 3$. In O ken ping gih /AB $\equiv 3x = 8y$; nai O' ken gih /AB yoo: $3x' - 8y' = 12$. Nai O ken, $A \equiv x = 8/\sqrt{5}$, $y = 3/\sqrt{5}$; nai O' ken, $A \equiv x' = 8/\sqrt{5} - 4$, $y' = 3/\sqrt{5} - 3$, hur yong x', y' in /AB, O' ken & yoo $24/\sqrt{5} - 12 - 24/\sqrt{5} + 24 = 12$; Q.E.D. Way Λ ping gih O' (x'",y'") ken meh.

Khaw O' chu & ken (x,y) hur $\Lambda \equiv 9x^2 + 16y^2 - 72x - 96y + 144 = 0$ & kay sia (kon TEK MEN (30 hao) $= 9(4-x)/16(y-3)$. Gih #A ($x = 4 + 8/\sqrt{5}$, $y = 3 + 3/\sqrt{5}$), $dy/dx = -3/2$. Gai maw a, mwa A, neh $(y + y')/(x - x') = S$ (Sia), hur chuh maw ping $\equiv 3x\sqrt{5} + 2y\sqrt{5} - 18\sqrt{5} - 18 = 0$ & yong y = 0, hur shee sup O'G"/ at $x = (10 - 6\sqrt{5})/\sqrt{5}$, or len x = 10.47·· meh. Chuh siao shik serve as review & preparation, mo kien kao sir.

(3) MIK SHIK LOON. Dang loon (5) siang khaw ℓ woo / & L chung gih gai yuan Λ & poo chung gih gwo (mo ching) kay tai pan kin LA = BL = 2 & siao pan kin LO = 1, hur Π (chuh gwo) ping $\equiv x^2 - 4y^2 - 8y - 8 = 0$. WAN kay mik? DAH.

Khaw y = SX + b shee mik ping, yong tung Π ping way:

$$x^2(1 - 4S^2) + x(-8Sb - 8S) - 4b^2 - 8b - 8 = 0.$$ Yong hee

(3 Hao) gih chang pun cher neh x^2 & x (x^n & x^{n-1}) tung ping

$(1 - 4S^2) = 0$ & $(-8Sb - 8S) = 0$, kay tung dah yoo boon neh $S = \pm 1/2$ & $b = -1$, yong tung mik ping: way

(I) $m \equiv 2y - x + 2 = 0$ & (II) $2y + x + 1 = 0 \equiv m'$. Shee kin gih ping harn hing ye kai = tai maw & yee kai = // tai kin ^{0} tung O & K.

45° Shik # W on II, neh G ($y = 2$, +45°

$x = 2\sqrt{10}$), D ($x = -2\sqrt{10}$, $y = 2$); H ($y = -2, x = 2\sqrt{10}$

$V(y=-2, x=-2\sqrt{10})$. Way so = $2\sqrt{10}$ mo kinen, neh = hyn gih ching kok sam kay ye tooy = 1 & 3, X 2 & yong Hjelmsleve fat wooy & hop tung tzu heng Zai gai yeet ping yong ung # yam II meh. Kon do fo kee.

YHJELMSLEVE

(4) TANG CHUNG YONG 2 GAI MIK SUP. Dang joy har AC = tai pan kin = $4\sqrt{5}$, CB = siao pan kin = $2\sqrt{15}$; chung C ken $\equiv$ $x = 6$, $y = -4$, hur tang E $\equiv$

2' 2' O

$(x-6)^2/80 + (y+4)^2/60 = 1$,

mik mik

or $3x^2 + 4y^2 - 36x + 32y = 68$ (E).

WAN gai kay chung ken; DAH yong sup gih kay mik neh $mm' = C$.

B Khaw m m' mik A $\equiv y = Sx + b$;

yong chuh ho tzu y tung E ping & yoo:

$3x^2 + 4S^2x^2 + 8Sxb + 4b^2 - 36x + 32Sx + 32b = 68$. Zhui ye chang x hee tzu joy $x^2(3 + 4S^2)$ & $x(8Sb + 32S - 36)$ & way ping neh: $4S^2 + 3 = 0$ & $8Sb + 32S - 36 = 0$, hur

$S^2 = \pm \sqrt{-3})/2$ & $b = (9 - 8S)/2S$. Yong chuh ho tung ($y = Sx + b$) & yoo: (1) $S = e/2$, $b = (9-4e)/e$, hur $y = ex/2 +$ B $\frac{9-4e}{e}$

& (2) $S = -e/2$, $b = (9+4e)/-e$, hur $y = -ex/e + (9+4e)/-e$. (Nim e joy = $\sqrt{-3}$ & $e^2 = -3$ meh.) Eliminate y by ping tzu (1) & (2) ho, hur

$e^2x + 18 - 8e)/2e = e^2x + 18 + 8e)/-2e$ & $e^2x = -18$, hur $x = 6$.

Yen (1) $y = -18 + 18 - 8e)/2e$ or (2) $y = -18 + 18 + 8e)/-2e$ hur $y = -4$ & Q.E.D. Thus by calculating the two ping for /m & m', which are both imaginary & then solving simultaneously these two equations we get the two coordinates of the real point C which is the center of the given ellipse. As an original exercise the good student will find the center of a circle using this formula & for practice do it in loon & lwen if you please. Do zai way chang ming pak & lik nim poo.

siao-pan-kin

O A — tai-pan-kin — $x = 6$ C $y = -4$

(5) KEN HIANG YI. Khaw Λ yeet ≡ $9x^2 + 16y^2 - 144$, tzu fat hing $Ax^2 + 2Bxy + Gy^2 + 2Dx + 2Hy + V = 0$, hur $A = 9$, $G = 16$, $V = -144$ & yur hee tzu = 0. Yong hee & ye fat neh

$E = AG - B^2$ & Δ = dun $\begin{pmatrix} A & B & D \\ B & G & H \\ D & H & V \end{pmatrix}$ hur joy $E = 144$ & $\Delta = -11556$.

Yeet fat: (1) if $\Delta = 0$ & $E = 0$, yeet shee ye // (chan kung) (2) if $\Delta = 0$ & $E \neq 0$, yeet shee ye + chih (chan gwo) (3) if $\Delta \neq 0$ & $E = 0$, yeet shee wot kung. (4) if $\Delta \neq 0$ & $E \neq 0$; yeet tang if E chang 0 & gwo if E sih 0.

Hur joy Λ tang yen $-11556 \neq 0$ & 144 chang 0.

WAN: wooy Λ, ting chu O, yi hiang gih kin neh yop Λ' whaw x' tzu & y' tzu y & kok $xx' =$ kok $yy' = \omega$; shik syn $\omega = 3/(\sqrt{13})$ & yn $\omega = 2/(\sqrt{13})$. Yong fat neh

$x = x' \cdot yn\ \omega - y' \cdot syn\ \omega$ &
$y = x' \cdot syn\ \omega + y' \cdot yn\ \omega$ in yi,
hur
$9/13(2x-3y)^2 + 16/13\,(3x+2y)^2 = 144$ &(leet)
$180x^2 + 145\,y^2 + 84\,xy = 1872 \equiv$ Λ'.

Fon yee, jak Λ' ping wan mo xy cher, hur wooy yong fat: tet $2\omega = 2B/(A - G) = (2 \text{ tet } \omega)/(1 - \text{tet}^2\,\omega)$

Hur joy 2ω tet $= 84/35$ & tet $\omega/(1 - \text{tet}^2\,\omega) = 42/35$. Khaw tet $= t$ & tet fong $= t^2$, hur dah ping $6t^2 + 5t = 6$ & $t = (\pm 13 - 5)/12$ or x'x kok tet $= -3/2$ or $2/3$, hur xx' tet $= 3/2$ or $-2/3$. Jak yang ho way xx' kwoo teng jawooy neh joy kok ω yoo tet $= 3/2$. Joy Λ' sup x,y ken $= \pm X = \pm \sqrt{(52/5)}$ = len 3.22 whaw y = 0 & $\pm Y = \pm 12\,(\sqrt{13})/\sqrt{145}$, neh len 3.59 whaw x = 0.

(6) CHU YI KEN HIANG TUNG. Dang lwen (II) WAN doong chu & mo yi ken hiang. Khaw ken gih sin chu O' ≡ x = m, y = n & gih A tung Λ, x = 4, y = 0, hur gih tzu Λ' tung sin yeet Λ' (tung ping), $x' = x + m = x + 5$ & $y' = y + n = y + 1$. Khaw Λ ≡ $9x^2 + 16y^2 = 144$, hur Λ' =
$9(x-5)^2 + 16\,(y-1)^2 = 144$ or
$9x^2 + 16y^2 - 90x - 32y + 97 = 0$.
Yam yong ken gih A' tung Λ' & yoo $729 + 16 + 97 = 810 + 32$.
Jeen yoo mo kien, nim shee mo yi hee gih chang kee cher tung ping meh. However one must practice assiduously constantly verifying the work to avoid error.

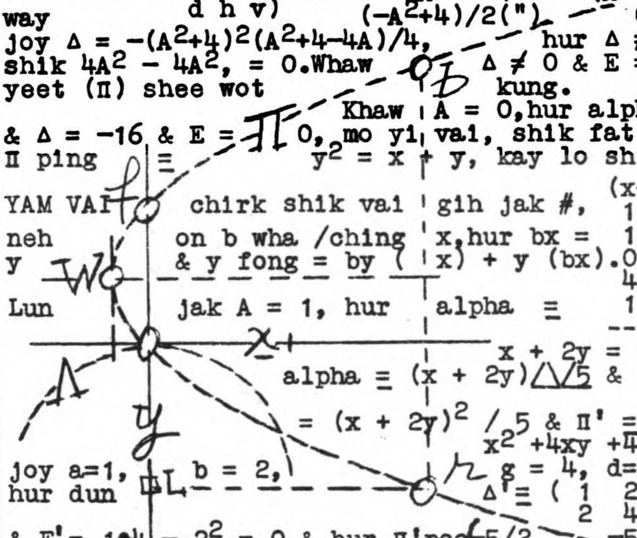
(7) LO WAN. Khaw alpha jak / on O (yong gai dang vai) neh shee sup Λ at O & jak # A lun, hur way OA (alpha) ping

(7 Hao) yong fat: A/B $\equiv$ $2(A+B)x + (4-AB)y = 2AB$, whawA & B ye # on Λ yuan. Joy $B = 0$, hur alpha $\equiv Ax + 2y = 0$. Gih # P tung meen, / chu P ching alpha kay so = PG, neh G shee gee on alpha gih /PG ching alpha, $= (Ax + 2y)/\sqrt{A^2 + 4}$.
WAN Lo whaw PG fong = x + y, (x,y ken gih P? DAH.
Chuh lo ping $\equiv$ PG fong $= x + y = (Ax + 2y)^2/(A^2 + 4)$, or

lo $\Pi \equiv A^2x^2 + 4Axy + 4y^2 - (A^2+4)x - (A^2+4)y = 0$, hur tzu

fat yeet: $ax^2 + 2byx + gy^2 + 2dx + 2hy + v = 0$, joy shik :
$a = A^2$, $b = 2A$, $g = 4$, $d = -(A^2+4)/2$, $h = -(A^2+4)/2$, $v = 0$

Nai dun: $\Delta = \begin{pmatrix} a & b & d \\ b & g & h \\ d & h & v \end{pmatrix}$ joy $\Delta = \begin{pmatrix} A^2 & 2A & -(A^2+4)/2 = (") \\ 2A & 4 & -(A^2+4)/2 \\ (-A^2+4)/2(") & & 0 \end{pmatrix}$. Leet way
joy $\Delta = -(A^2+4)^2(A^2+4-4A)/4$, hur $\Delta \neq 0$. $E = ag - b^2$,
shik $4A^2 - 4A^2, = 0$. Whaw $\Delta \neq 0$ & $E = 0$ (kon (5))hur yeet (Π) shee wot kung.



Khaw A = 0, hur alpha = x (maw mwa 0) & $\Delta = -16$ & E = 0, mo yi vai, shik fat. Shee A = 0 hur Π ping $\equiv$ $y^2 = x + y$, kay lo shik tung joy dang.

YAM VAI chirk shik vai gih jak #,
neh on b wha /ching x, hur bx =
y & y fong = by (x) + y (bx).

(x+y)	#	x	y	y^2
1/4	W	−1/4	1/2	1/4
1	f	0	1	1
0	0	0	0	0
4	b	2	2	4
1	r	2	−1	1

Lun jak A = 1, hur alpha $\equiv$ $x + 2y = 0$ & PG ching alpha $\equiv (x + 2y)/\sqrt{5}$ & PG fong = x + y
$= (x + 2y)^2/5$ & $\Pi' \equiv x^2 + 4xy + 4y^2 - 5x - 5y = 0$;
joy a=1, b = 2, g = 4, d= −5/2, h=−5/2, v=0
hur dun $\Delta' \equiv \begin{pmatrix} 1 & 2 & -5/2 \\ 2 & 4 & -5/2 \\ -5/2 & -5/2 & 0 \end{pmatrix} = -25/4$
& $E' = 1\cdot 4 - 2^2 = 0$ & hur Π' poo wot kung.

Chirk yam: x y (x + 2y)fong = 5(x + y)
shik- 0 5/4 100/16 = 25/4
$x = (1 \pm \sqrt{5})/2$ y=1 $10(3 \pm \sqrt{5})/4$ = $5(3 \pm \sqrt{5})/2$ ju chuh.

Neh PG fong $= (x + 2y)^2 = 5(x + y)$, gih Π' ; & Q.E.D.

(8) LO SHEE GWO. Yong x & y ken - ching mo fat chan, ye / ting & C jak ting meen #. Yong hung C way but, kay / sup x & y, as ken gih # P. Wan P lo? DAH. Jak / alpha, sup x, y = T, Q, shee ken gih P, neh P ken $\equiv$ x = T & y = Q. Ken gih C $\equiv$ x = a, y = b. Gih ju sam TPQ & CaT kon ping fun: TP / Ca = PQ / aT. Yong x,y = P ken. TP = y, PQ = −x, Ca = −b, aT = −(a −x), whaw a,b $\equiv$ C ken. Hur y/−b = −x/−(a −x) or <u>ay + bx = xy</u>, shee ping gih P lo, neh Π.

Nim Π yeet yen

C tzu	x tzu	y tzu	M tzu	W
alpha	T	Q	y	x
alpha'	T'	Q'	y'	x'

hur x & y (x' & y') ye tzu ray gih ye chuan but kay hung W & M (woo #).

(8 Hao) Wan π thi? Yong dun Δ & E fat neh: gih π ping, A=0, B = −1/2, G = 0, D = −b/2, H = −a/2, V = 0, hur Δ (A B D B G H D H V)

(0 −1/2 −b/2 = ab(1−b)/8 ≠ 0 & E = AG −B²
Δ= −b/2 0 −a/2
−b/2 −a/2 0) joy E = −b²/4 ≠0, neh sih 0,
hur yeet wot gwo. P
Wan kay M mik & chung?
DAH yong fat(41 Toeng DU) neh ken gih chung C ≡
x = (GD −BH)/(B² −AG) & y = (AH −BD) /(B² −AG) shik
x = 0−(a/2· 1/2)/(− 1/2)² −(0) = (a/4)/(T 1/4)
W x T a
& x = +a. y = (0−(− 1/2· b/2)/(− 1/2)²− 0,
or (b/4)/(1/4)& y = b; hur ken gih π chung ≡
ken Λ gih C, neh, x= Λ a & y = mik b, dang joy.
Gai mik mik yong fat (1)neh khaw maw
b y = Sx + c,
sup π & if sup tung woo /,hur xⁿ & xⁿ⁻¹ hee = 0.
Yong (Sx + c) tzu (y) nai π ping & yoo:
a(Sx + c) + bx y − x(Sx + c) = 0 or
aSx + ac + bx −Sx² −cx = 0 neh
x² hee = (−S) & x hee = (aS + b −c); chien way ye ping:

−S = 0 & aS + b = C, hur dah tung way S= 0 & c = b, hur
y = Sx + c tzu y = b shee yot mik. Lun mik ≡ x = a,
hur ye mik joy // ken & tung C.
Meen YAM: yong Ung #
Sing Maw Fat gih ung π #,shik: (M P + P'O)/(MP' + P W),+
OW = a on x,hur Ma = MC = π maw mwa M, yot mik. Ju:
(WO + PP')/(WP + OM) + P'M,/W = WCb = lun mik meh.

Lun way gai C chung yong fat: Ax + By + D = 0 tung
Bx + Gy + H =0 neh joy G = 1,D = b/2,H = a/2, P' hur
(B = −1/2) −1/2 y + b/2 = 0 or y = b & −1/2x)+a/2 = 0,or
x = a; Q.E.D. Zai way tzu loon & pee yong tsia ken meh.

Nim P tung π lo yoo ken x = T & y = Q. If Q be in terms of alpha as E/, then y = 2q/(4 −q); T = q/p & joy T = 2Q/Q+1 or Q = T/2−T; q = 4(p−1), ju chuh. In terms of a & b, t = (a + 2b)/(b + 1), hence indeterminate from a & b alone Of the hyperbola, y = x/(2 −x), the center,C as derived from B = 1/2, D = 1/2, H = −1, yielding x = 2 & y = −1. π joy is connectedwith a family of degenerate cubics composed of the two fixed / x= 0, y= 0 & the variable /alpha always through fixed # C. For any alpha or E/ there is a unique # C & ion yee & either can be deduced when lun jak. Different methods of doing this are suggested here. Kao sir way lun gai,nih kih,zai & yoo ming pak meh. Shik write P in terms of A & B as Λ intercepts of E/ on C & vary #C. Test for singularities,exceptional #,/ etc.

(9) WAY MAW GWO LO LOON SHIK. Yong chung C ken $a = 2$, $b = -3/2$ & E/ on C sup Λ at $A = 2$, $B = -2/3$, hur Ey (q) = $-4/3$ & Ex (t) = $-1'$, poo Ek (P") = $A+B = p = 4''/3$ meh. Khaw gwo lo = Gamma, shik #G kay ken $x = Gt$, $t = -1'$ & $y = GqY$, neh $y = -1/2$. Yen $t = q/p$, G yoo $x = AB/(A+B)$ & $y = 2q/(4-q) = 2AB/(4-AB)$ Kon Tai Shu TUI HOI (980). Gamma $\equiv ay + bx - xy = 0$.

Gih jak #G on Gamma hur $2aq/(4-q) + bq/p = 2q^2/p(4-q)$ & hur $2ap + b(4-q) = 2q$ or $2a(A+B) + b(4-AB) = 2AB$.

Shik joy, $a = 2$, $b = -3/2$ hur $8(A+B) - 3(4-AB) = 4AB$ or $8(A+B) - AB = 12$ or $\underline{8p - q = 12} \equiv$ Gamma (G lo=gwo).

WAN way Gamma maw mwa G. Shik ye way (I) GEEK WAY. Fat yeet ping $\equiv Ax^2 + 2Bxy + Gy^2 + 2Dx + 2Hy + V = 0$, tzu fat geek ping hing $\equiv Axx' + B(xy' + x'y) + Gyy' + D(x+x') + H(y+y') + V = 0$, Hur joy $bx - xy + ay = 0$ tzu $2Dx+2Hy+2Bxy = 0$, hur $D = b/2$, $H = a/2$, $B = -1/2$ & $D(x+x') + H(y+y') + B(xy' + x'y) = 0$, tzu $b(x+x')+a(y+y')-(xy'+x'y) = 0$ Yen $a=2$ & $b=-3/2$ shee $-3(x+x')+4(y+y')-2xy'-2x'y = 0$ gih jak # G & yen G ken joy $\equiv x' = -1$ & $y' = -1/2$, hur maw g $\equiv$ $-3(x-1) + 4(y-1/2) + x + 2y = 0$, or $\underline{2x - 6y = 1}$.

(II) SIA WAY. Implicit differentiation of $ay + bx - xy = 0$ gives $dy/dx = (y' - b)/(a' - x)$, shee slope of lwan gamma at # kay ken $\equiv x', y'$, hur gih G ($x'=-1$, $y' = -1/2$) yoo: $-(2b+1)/(2(a+1))$. Maw fat $\equiv (y - y')/(x - x')$ = Sia, hur joy gih G: $(2y + 1)/2(x+1) = -(2b + 1)/2(a+1)$ & dah yoo: maw tzu mwa G (C (a,b) $\equiv \underline{(2y+1)/(x+1) = -(2b+1)/(a+1)}$.
Shik a= 1, b= $-3/2$, hur $(2y+1)/(x+1) = -(-3+1)/(2+1)$, or $2x - 6y - 1 = 0$; Q.E.D.
Yen $x = 0$, hur $y = -1/6$ & $y = 0$, $x=1/2$, ——— maw g ken sup way kay wha mo kien.

MEEN YAM Way lun Gamma # neh P (shik $x = 1$, $y = 3/2$) & yong Ung # Sing Maw Fat, tung G, O (xy), Y (xℓ), W (yℓ), & P. Neh, (GO + YP)/(GY + OW), sup PW, lyn G, = g maw, Q.E.F.

Nim poo (gih Gamma) $dy/dx = \dfrac{(b(4-AB) - 2AB)(A+B)}{(4-AB)((AB - a(A+B))} = S$,

$g \equiv \dfrac{((4-AB)y - 2AB)(A+B)}{(4-AB)((A+B)x - AB))} = S$, hur tzu C yoo ken $a=2$, $b=-3/2$

$g \equiv ((4y - AB(y+2))(AB - 2A - 2B) = (Ax + Bx - AB)(-12 - AB)/2$, or,

yong $A = 2$, $B = -2/3$: leet shee $(2x - 6y = 1) \equiv g$; Q.E.D.

(10) VAI LO YONG LUN KEN FAT, neh x = x(tzu A), y= y(tzu B)

(10 Hao) khaw A chang B. Nim shee A chang B hur (A − B) +, if A sih B, then (A − B) is minus, yen jak yin ho sih ling neh −∞ sih 0, ju chuh. Joy K = + ∞ on Λ & ju 0 = ± 0. (Kon MAR SIANG (8,9,10).

Khaw ting # C, x=a, b=y, shik (1,1). Chih tung C sup Λ at G (chang) D; WG + DY = H on lo Eta. Geek tzu keek C + Λ joy at 2 & −2/3, shee poo tung Eta & Λ chung L on lo Eta. Hook deem shik lo, ye gyih, mo yeet, chang pun lwan; keen twan gih lwan yoo kwing (stationary) # shik −2/3, 2 & ye lun whaw lwan mo hao joy.

Way kay ping: yen ju sam $b/\underline{a-t}$ = $b(G+D)/(a(G+D)-GD) = HG/DH = (y_G - y_D)/(x_G - x_D)$, or

Eta $\equiv 2a(G+D) + b(4 - GD) = 2GD$. Yen $x = 4G/(4+G^2)$ & $y = -2D^2/(4+D^2)$ nim $(4+G^2) = 4G/x$ & $(4+D^2) = -2D^2/y$ hur

$b(G+D)/\underline{a(G+D)-GD} = D(2y+Gx)\,/\underline{-(2y+Dx)}.$

khaw a = b = 1, hur

$(G+D)/\underline{GD-G+D} = D(2y+Gx)/(2y+Dx).$

Shik yong G=0 D= −2, hur x= 0, gih # L.

(11) SHIK. Yong C = L, neh a = 0, b = −1 & way lo Eta Zo. Tsih yong ju Δ, Λ hur $b/\underline{a-t} = HG/DH = GH/HD$ & hur

$b(G+D)/\underline{a(G+D)-GD} = (y_D - y_G)/(x_D - x_G)$ or

$2a(G+D) + b(4 - GD) = 2GD$. Dang (11) a = 0, b = −1, hur Eta Zo lo $\equiv$ GD = −4, neh chun / on Λ chung joy C, sup Λ at G chang D kay shing =(−4).

YAM joy Eta Zo = Λ kay ping $\equiv x^2 + y^2 + 2y = 0$.

Yen $y = -2D^2/\underline{4+D^2}$, hur $D = -\sqrt{-4y}\,/\,\underline{\sqrt{y+2}}$ (use the minus sq. root) & yen $x = 4G/\underline{4+G^2}$, (for G use the plus square root) $G = \dfrac{2+\sqrt{\underline{4-4x^2}}}{x}$

Yong chuh G & D ho nai ping GD = −4 & yoo:

$(2+\sqrt{\underline{4-4x^2}})/x \times \sqrt{\underline{-4y}}/\underline{\sqrt{y+2}} = 4$, shee leet

yop $2(x^2+y) = -y\sqrt{\underline{4-4x^2}}$, or

$x^4 + x^2y^2 + 2x^2y = 0$ & $\underline{x^2+y^2+2y=0}$;

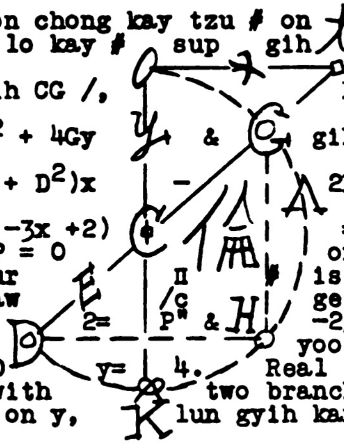
hence when C is taken as center of gai yuan Λ & Eta Zo lo is locus of # whose coordinates are x is the x of G & y is the y of D where G greater than D are intersections of lines of the pencil centered at C, curve generated in this fashion is the circle Λ itself = Eta Zo. The equation can be simplified further as $\underline{q + 4 = 0}$, the general form for any C# $\equiv$ 2ap + b(4 − q) = 2q: (G chang D).

(12) HOOK LO SHIK. Dang har joy yong lwen gai yuan Λ & jak # C (1,1). E/ on C,+ Λ = G chang D. Jak way # on Λ = Z, shik 4 joy. Hur yong O,lun Λ way # (sih kien gai) & O/ sih Λ# neh D to sup Z / chang Λ# neh G at P tung lo Π;neh OD + ZG = P. Wan Π ping? DAH. Fat ping gih ye Λ # shik: (Shik O = O & Z = 4 joy)

O/D ≡ 2(O+D)x + (4−OD)y = 2OD, or Dx + 2y = 0. Ju:
Z/G ≡ 2(4+G)x + (4 −4G)y = 8G, hur, dah tung way:

x(G − D + 4) = 2G(y + 2) or G(x−2y−4) = x(D −4) ≡ Π joy.

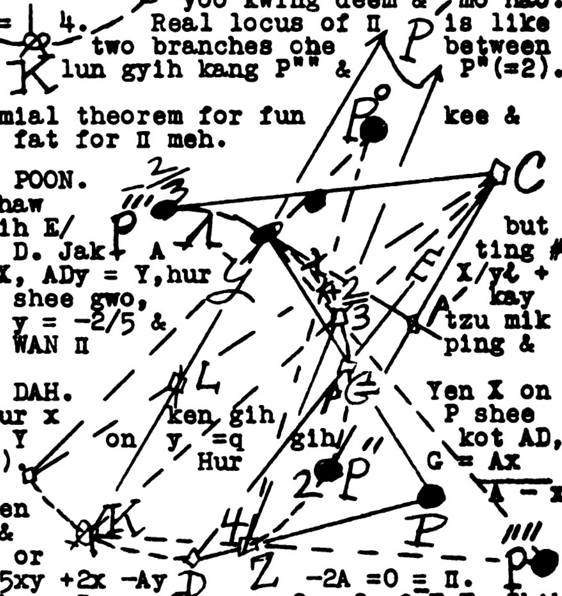
Nim shee C poon chong kay tzu # on Λ, / O & D as ye sin hung way Π as lo kay # sup gih ye chuan but meh.



Way E ping: gih CG /, $E \equiv y(4+G^2)+4G - 2G^2x = (4+G^2)x - 2G^2 + 4Gy$ & gih CD / $E \equiv (4+D^2)y + 4D - 2D^2x = (4+D^2)x - 2D^2 + 4Dy$ hur

E ≡ (G + D)(y −3x +2) = 4(y −1). If G = 0, D = −2, then P = O on Λ. If G = 2/3 D = K. Jak K = −∞, hur Π # is P'; jak K = +∞ woo,hur Π # = P"". Khaw /C geek tzu C keek,sup Λ at 2 & −2/3,hur 2= P" & −2/3 = P'" tung Π lo. Lwan lun shik yoo kwing deem & mo hao. Nim P' kay x=O y= 4. Real locus of Π is like an hyperbola with two branches one between P'" on Λ & P' on y, lun gyih kang P"" & P"(=2).

Nim yong fat gih binomial theorem for fun kee & tzu sia gai & yoo maw fat for Π meh.

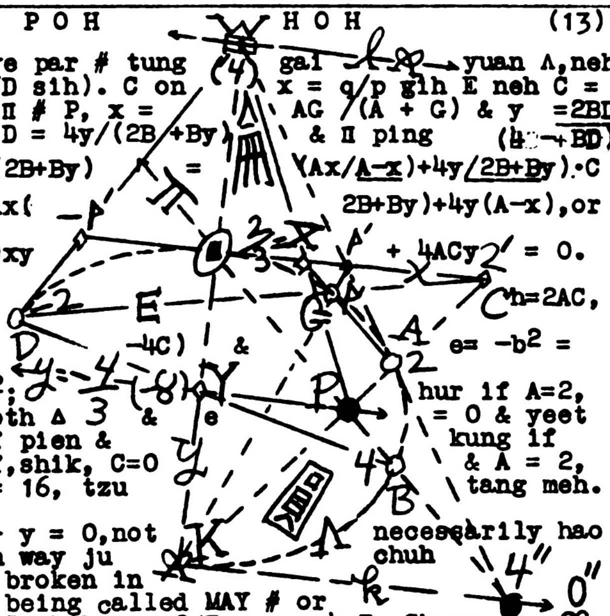


(13) GWO LO TZU SHING POON.
Dang (12) lwen khaw ting # C on y chung gih E/ but kay / sup Λ = G chang D. Jak Λ ting # on gai yuan Λ. AGx = X, ADy = Y,hur X/yℓ + Y/xℓ = P # tung lo Π, shee gwo, kay chung Pe,neh x = 1/5, y = −2/5 & tzu mik on Pe,ching & // x,y. WAN Π ping & wha maw mwa P, neh p.

DAH. Yen X on x = q/p gih kot AG, hur x ken gih P shee AG / (A + G) & ju,yen Y on y =q gih kot AD, P ken y = 2AD /(4 −AD). Hur $G = \frac{Ax}{A-x}$ & D = 4y /(2 + y)A. Yen GXD = 1, G & D siong & Ax/(A− x)= A(2+y)/4y, or 4xy = (A −x)(2+y) or 5xy +2x −Ay −2A =0 ≡ Π. Shik A = 1, hur Π shik ≡ 5xy + 2x − y −2 = 0; Q.E.F. Shik #: G=K,D = 0,hur for P',x=1',y= 0; if G=0,D=K, x=0,y=−2, & P" = K; if G=D=−1, x= xℓ,y= −2/5;if G=D=+1,x=1/2,y= 2/3;if

(13 Hao) G =-1/4, D = -4 & P ken, x= -1/3, y=-1.
Tzu fat ping $Ax^2 + 2Bxy + Gy^2 + 2Dx + 2Hy + V = 0$
(yong siao chee), Π = $2bxy + 2dx + 2hy + v = 0$ &
b = 5/2, d= 1, h = -a/2 & v = -2a. Hur $\Delta\begin{pmatrix} a & b & d \\ b & g & h \\ d & h & v \end{pmatrix}$
neh joy Δ=10A ≠ 0 & $e = ag - b^2$ or
e = -25/4 shee sih ling, hur Π gwo.
Shee wot.
Π chung yong fat ax + by + d = 0 &
bx + gy + h = 0, hur ken gih
chung Pe: x=1/5 &
y= -27/5. Yong mik fat neh put
y = Sx + c in Π ping &
equate hee of jeet x^2 cher to ling &
ju hee ping gih x zhui cher, shik x(5b +2 -AS)
& gai peh S = 0 & c = -2/5; ju chuh. Ye
mik sup tung gwo chung, Pe. Lun gai
chung : $x_2 = (gd - bh)/(b^2 - ag)$ & ken
$y = (ah - bd)/(b^2 - ag)$; kon TOONG DU, twan 41)
SHIK. Yong fat(y - y')/(x -x')=S.
S =sia = dy/dx = (5y +2)/(A -5x) shik (5y+2)/(1 -
5x) whaw A=1. Hur gih #P (x=-1/3, y = -1) sia =
-9/8 & maw p = 9x + 8y + 11 = 0. Yong
x = 0 & gai y sup neh -11/8 & tung y = 0
gai x sup neh -11/9, mo kien wha maw p.
Poo khaw yong meen fat neh Ung # Sing Fat meh.
Nim S = XP/TX, if T = px, hur joy gih yong P, XP/TX fun
= & tzu 8/-9, neh TX yang& XP yin so joy. Chuh tsen meh.
Nih pee chuh fo & booy fo (8) way chang ming pak.

(14) GWO LO TZU KAR POON. Dang joy har khaw C tung kar
maw k, hur fat (G + D = C), joy shik C = 2, G=
4, D = -2. G chang D. G & D lyn jak Λ # A. AGx = X, ADy =
Y. X/yl sup Y/xl = P, hur ken gih Π # P ≡ x = AG/(A+G) &
y = 2AD/(4 -D). WAN Π ping. DAH. Yen G + D = C yong kay
ho tung ping neh: $G = Ax/(A - x)$ & $D = \frac{4y}{(2A + y)}$.
Hur $\frac{Ax}{A - x} + \frac{4y}{2A + y} = C = 2$ or
$2Ax(A + C) + Axy(A - 4 + C) + Ay(4 - C) - 2A^2C = 0$
(A=1,C=2) ≡ joy Π ≡ 6x - xy + 2y -4 = 0;Q.E.D.
Mo kien gai Δ ≠ 0 & e sih 0, hur Π wot gwo.
Gai chung yong ax + by + d= bx + gy + h = 0 &
hur ken gih chung Pe ≡ x = 2A/2-A y=6A/2-A
meh. Way maw yong fat y-y' = S(x -x')ju
chuh.
(15) GWO LO TZU TOK POON. Dang loon (13
shik Π chan yeet, yong C = 2' on tok
maw x, A = 2 on gai yuan Λ, B= 4
on Λ, neh A, B, C shee sarm par
& / on C sup Λ = G chang D, hur
AGx = X; BDy = Y, neh P ken; WX + xl/Y = P # tung lo Π.



(15 Hao) Lo yong ye par # tung gai yuan Λ, neh A (/G chang) & B(/D sih). C on $x = q/p$ gih E neh $C = GD/(G+D)$. Gih jak Π # P, $x = AG/(A + G)$ & $y = 2BD/(4 + BD)$ hur $G = Ax/(A - x)$; $D = 4y/(2B + By)$ & Π ping

$$= \frac{(Ax/(A-x))(4y/(2B+By))}{(Ax/(A-x)) + 4y/(2B+By)} \cdot C$$

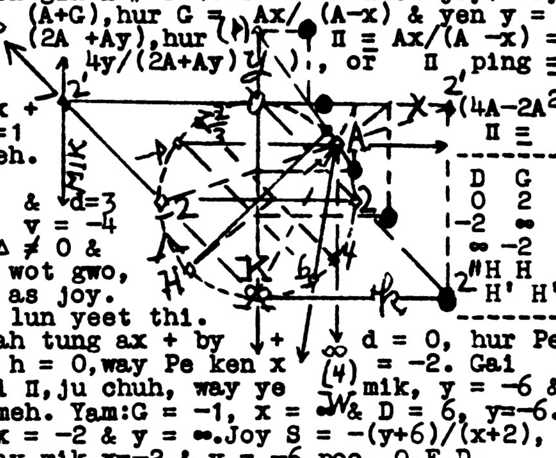
or $C = 4Axy/(Ax(2B+By)+4y(A-x))$, or

$$2ABCx + (ABC-4C-4A)xy + 4ACy^2 = 0.$$

Hur $b = \frac{(ABC-4C-4A)}{2}$, $h=2AC$, $d=ABC$ & $\Delta = 2A^2BC^2(ABC-4A-4C)$ & $e = -b^2 = -(ABC-4C-4A)^2/2^2$; hur if $A=2$, $B = 4$, & $C = 2$, both Δ & e = 0 & yeet shee //, neh gwo if pien & kung if tung, both chan. If, shik, $C=0$ & $A = 2$, $B = 4$, $\Delta = 0$ & $e = 16$, tzu tang meh.

Dang shik $\Pi = 2x + y = 0$, not necessarily hao chuh (continuous). Lwan way ju khaw twan tsing & broken in places, such ends being called MAY # or "point d'arret". P shik $x = 2/3$, $y = -4/3$. Chung gih Π yoo ken $x = 4AC/(ABC-4C-4A)$, $y = 2ABC/(ABC-4C-4A)$, or $x = \infty$, $y = \infty$, joy. Zai yong lun ho gih C, poo A & B, meh.

(16) LO YONG NEE GAI. Dang lwen har khaw $C = -45^\circ$# gih TTP tung woo / ℓ, hur chun E chih // & C chong gih poon on Λ kay tzu # G & D. Yong nee gai: $(0\ -2\ \infty\ D) = (2\ \infty\ -2\ G)$, hur $D = (4-2G)/(2+G)$ & $G = (4-2D)/(2+D)$ & hai ##, H, (zo), $= (\pm 2\sqrt{2} - 2)$. Jak # A on Λ (shik = 1); $AGx = X$, $ADy = Y$; X & Y shee x & y ken gih Π # P (shik hook deem joy). Way Π ping: yen $x = AG/(A+G)$, hur $G = Ax/(A-x)$ & yen $y = 2AD/(4-AD)$, $D = 4y/(2A+Ay)$, hur $\Pi \equiv Ax/(A-x) = (4-8y/(2A+Ay))/(2+4y/(2A+Ay))$, or Π ping ≡



$(A^2+4A-4)xy + (2A^2+4A)x + (4A-2A^2)y - 4A^2 = 0$ & shik $A=1$ Π ≡ $xy + 6x + 2y - 4 = 0$ meh.

Shik, $\Delta = \begin{vmatrix} 0 & 1/2 & 3 \\ 1/2 & 0 & 1 \\ 3 & 1 & -4 \end{vmatrix} = 8$ & $d=3$, $b=1/2$, $h = 1$, $v = -4$ ($e = -b^2 = -1/4$), hur $\Delta \neq 0$ & e sih ling, hur yeet Π wot gwo, if par ($A=1$, $C = -45^\circ$) as joy.

D	G
0	2
−2	∞
∞	−2
##H	H
H'	H'

Lun par shik khaw way lun yeet thi.

Gai chung yong fat: dah tung $ax + by + d = 0$, hur Pe ken $y = -6$; $bx + gy + h = 0$, way Pe ken $x = -2$. Gai mik put $y = Sx + c$ nai Π, ju chuh, way ye mik, $y = -6$ & $x = -2$, tung chung Pe meh. Yam: $G = -1$, $x = $ & $D = 6$, $y=-6$. Poo, $D=4$, $G = -2/3$; hur $x = -2$ & $y = \infty$. Joy $S = -(y+6)/(x+2)$, hur $(y+y')/(x-x') = S$, way mik $x=-2$ & $y = -6$, poo. Q.E.D.

(17) LO TZU SUP FUN C YOO. Khaw C joy ken x= −1,y= −5/3.
Shik E/G = 2, D = −4 & khaw A = 1, hur P = x = 2/3,
y = −1. Tung A poon, C chong, #G tzu #D, hur yong nee gai:

(G 2 −2 −4) = (D −4 −2 2) or G = (−6D −20)/D +6
& D = (−6G −20)/(G + 6) & hai # U,V = ± 4 −6, neh
U = −2 & V = −10. x = AG/A + G
& y = 2AD/4 −AD hur G = Ax/(A − x) &
D = 4y/(2A + Ay), hur $\frac{Ax}{A-x}$ = $\frac{(-12y -20A - 10Ay)}{2y + 6A + 3Ay}$ &
(8A−3A²+12)xy +(20A−6A²) x − (12A + 10A²)y −20A² = 0,
hur shik A = 1, Π lo = 17xy +14x −22y −20 = 0;QEF.
Yam yong x = 2/3,y=−1 way ping ling meh.

Gih chuh Π shik: Δ = 136 & e = −289/4,whaw d=7,
b = 17/2 h= −11 v = −20. Hur Π wot gwo.

Gih chung Pe: yong ax +by +d =0
& bx + gy + h = 0,hur ken y = −14/17 &
x = 22/17, shee poo ping gih ye mik meh.

Lun way mik: wha /A −1,+ x = ∞; C/−1,+ A,
lyn A,+ y = −14/17. Ju, A/4,+ y = ∞;4/C +
A,/A,+ x = 22/17 meh.
Yam yong toy gai:if x = ∞
then 17 ∞y + 14 ∞ −22y −20 = 0 &
y(17∞ −22) = (20 −14∞),hur y = $\frac{(-14\infty +20)}{17\infty -22}$
or y = −14/17. Ju if y = ∞, 17∞x+14x−22∞−20= 0 &
x = (22 ∞ + 20)/(17∞ +14) & hur x = 22/17; Q.E.D. If you
are careful in handling quantities like 0 & ∞ in gai
center of any conic can be got by partial differentia-
tion;e.g. with Π = 17xy + 14x −22y −20 = 0,the partial
derivative of Π gih x = 17y + 14 = 0, or y = −14/17 & if
x be held constant & Π differentiated gih y,we get 17x −
22 = 0, or the ken of x is 22/17. Jak lun shik,wan chung
gih yuan A,neh = x² + y² + 2y = 0. By siang way 2x = 0 &
2y + 2 = 0, or ken gih chung L = x = 0, y = −1; Q.E.D. In
case we forget the formulae (ax + by +d=0) &(bx +gy +h=0)
which,by the way, have been developed as general express-
ions using the same partial sia fat, we can use the simple
method as above which got these formulae,without bothering
to find a,b,g,etc, the coefficients from the fat yeet hing
however the Δ & e fat are useful to determine yeet thi,but
for yeet chung only the twan sia fat is easiest. Zai lun.

(18) LO TZU FAT CHUAN TUNG GAI YUAN A. Dang (15) lwen C mo
deem,neh chan feng, joy shee wot feng yen chun E lyn
mo yoo yot sup; fon yee GD maw wot yeet, shee lyn tzu #
gih chuan, shik kon chirk, hur (2 0 −2 G) tzu & G D
= (−1 0 −2 D) or (−2G−4)/(G−2)=(D +2)/(D+1) & 0 0
3GD + 4G = −2D or 3GD + 2D + −4G & G =−2D/(3D+4) −2 −2
while D = −4G/(3G + 2), mo poon. Gai hai # neh: 2 −1
3G² +2G =−4G or U,V = (± −1) neh U = 0,V= −2. -------

(18 Hao) Yen x = AG/A+G & y = 2AD/4−AD G = Ax/A−x & D = 4y/2A + Ay, hur yong chuan yi ping & yoo:

$$\frac{Ax}{A-x} = \frac{(-8y/(2A+Ay)}{(12y/2A+Ay)+4}$$

or Ax/A−x = −2y /(3y + 2A + AY), hur

Π ≡ xy $(3A + A^2 -2) + 2A^2x$ + 2Ay = 0,or xy + x + y = 0 if A = 1. Ju way gai tung ping yong 4y/2A+AY = −4Ax/A−x / 3Ax/A−x + 2.

Gih xy + x + y = 0, b = 1/2, d = 1/2 h=1/2 & yur hee = 0, hur Δ = −1/2(−1/4 2 −1/4) = 1/4 ≠ 0 neh dah dun (abd & e = ag / − b^2 = −1/4, hur Π wot gwo bgh Way chung & mik:

0	1/2	.5
.5	0	.5
.5	.5	0

yong dhv)

ax+by+d=0

bx+gy+h=0 hur

y/2+1/2=0;x/2+1/2=0 & ken ≡ x = −1, y = −1,neh ≡ −2 on Λ. Gai mik: put Π hur Sx^2 + cx + x + Sx + c = 0 & x hee = (c + 1 + S) = 0, hur yot mik ju chuh. Lun gai: if x = ∞ ∞y + ∞ + y= 0 & y = −∞ / (∞ + 1) if y = ∞, Π ≡ ∞x + x + ∞ = 0 & x = −∞/∞ +1 mik meh. | AGx = X ; ADy = Y ; hur D = ∞ = K. A/G = 2(A +G)x + (4− AG)y = 2AG, hur A/G ≡ x + 7y = −2 & if y = 0, x = −2 gih P. Wan y gih chuh P. Dah. Yen G= −2/3 x=AG/A+G way x = −2 poo gih P & D = −4G/3G ± 2) way D= ∞=K & y = −2 gih P. Om put x = −2 nai Π ping & yoo tung neh y =−2. Tung meen wha AG sup x = X & AD = AK,+ y=Y; hur X/yℓ + Y/xℓ = P joy.

Pe = y= Sx + c nai x(c+1 +S) shik shee y=c = −1; Π ≡ or $\bar{y}$ = −1. Ju, lun shik G = −2/3,

gih Pe V

Ju way mik: A//y + Λ = 4 & + y = ∞; yong D = 4, hur G = −8/(12 +4) = −1/2 & yen x = AG/A+G x joy = x =−1. Ju, wha chih tung A // x, neh A−1 shee A/G,sup x = ∞,hur if x = ∞, G = −1 & D = 4/(−3+2) = −4 & y =−8/4+4 = −1. Way maw gih Π # P shik (−2.−2 joy);yong fat gih sia S = (y −y')/(x −x'). Gai sia gih Π by implicit differentiation neh S = −(y+1)/(x+1),or for P,S = −1, hur maw p=x+y+4= 0, hur kay axial sup ≡ x = −4 & y = −4, mo kien wha meh. Zai.

(19) FAT WAY TANG SHIK. Wha gai yuan Λ (x^2+y^2+2y =0) & jak ye #,shik A = 4, B = −2, on Λ. Khaw jak so on Λ tzu s ,shik, = 1, = M − N, neh yong K as hung hop x tok to Λ,hur on x, M' chang N' & M' kam N' = jak so s = 1, & on Λ arc MN tzu twan MN on x. Thus the straight distance MN on x is projectively equal to the curved so MN on Λ, although of course not metrically ping. Chuh so shee jak,neh shik MN = 2, 3, ju chuh, joy shik MN = −1, NM = +1, sen. So(M −N)can wander around the circle, the distance itself is what is fixed (ting); each particular # P of lo Π tzu unique interval MN on Λ. Dang (16) shik ye #P & P',hur(M − N)tzu P, =1 & M'−N',tzu P', = 1,poo,neh tzu P, 1 −0 = 1 & tzu P',M'−N'

(19 Hao) corresponds to $(-4-(-5)=1)$. Way P: AM sup BN=P, or AM' + BN' = P', etc. The parameters A & B are fixed for any particular curve, while M varies, N being a function of M, or fon yee, the real par in this case being the so $M - N = 1$, which also is ting for shik lwan. Gai Π ping: $AMP = 2(A+M)x + (4 - AM)y = 2AM$. Shik $A = 4$, hur $2(4+M)x + (4 - 4M)y = 8M$, or $M(2x - 4y - 8) = -8x - 4y$.

Ju, $BNP = 2(B+N)x + (4-BN)y = 2BN$, or shik $B = -2$, hur $2(-2+N)x + (4+2N)y = -4N$ & $N(2x+2y+4) = 4x - 4y$. Yen $M - N = s = 1$ $(-8x-4y)/(2x-4y-8) - (4x-4y)/(2x+2y+4) = 1$, shee Π ping. Kay leet : $4x^2 + 4xy + 8x + 2xy + 2y^2 + 4y + 2x^2 - 4xy - 8x - 2xy + 4y^2 + 8y + x^2 + xy + 2x - 2xy - 2y^2 - 4y - 4x - 4y - 8 = 0$, or $7x^2 - xy - 2x + 4y + 4y^2 - 8 = 0$ shee Π. Yam yong P ken $(x=4/5, y=4/5)$ e.g.

Gai Π thi;tzu fat hing: $ax^2+2bxy+gy^2+2dx+2hy+v = 0$, joy $a = 7$, $b = -1/2$, $g = 4$, $d = -1$, $h = 2$, $v = -8$, hur $\Delta = \begin{pmatrix} a & b & d \\ b & g & h \\ d & h & v \end{pmatrix}$ becomes $\begin{pmatrix} 7 & -1/2 & -1 \\ -1/2 & 4 & 2 \\ -1 & 2 & -8 \end{pmatrix} = -252 \neq 0$ & $e = ag - b^2 = 111/4$ chang ling, hur Π wot tang meh. Gai chung: twan sia gih Π = $14x - y = 2$ & $-x + 8y + 4 = 0$, kay dah tung way $x = 12/111$ & $y = 54/111$, or chung gih Π = Pe kay ken: $x = 4/37$ & $y = -\frac{18}{37}$.

Yam yong fat $ax+by+d=0$ tung $bx+gy+h=0$. Tung meen nim gih $14x - y - 2 = 0$, khaw $y = 0$, hur $x = 1/7$ & khaw $x=0$, $y = -2$, hur mo kien wha & ju gih $-x + 8y + 4 = 0$, kay axial sup are $(x=0, y=-1/2)$ & $(y=0, x=4)$.

Shik way chung as sup gih ye mik (xing ping): put $y = Sx + c$ in Π & collect & equate to zero coefficients of two highest degrees of x, neh $(7 - S + 4S^2)x^2$ & $(-c + 8Sc - 2 + 4S)x$, hur $4S^2 - S = -7$ & $S = (1 + \sqrt{-111})/8$ tzu $c = (3 - \sqrt{-111})/(2\sqrt{-111})$, hur yot mik ≡ (1) ≡ $y = (1+\sqrt{-111})x/8 + (3-\sqrt{-111})/(2\sqrt{-111})$ &
(2) $y = (1-\sqrt{-111})x/8 + (3+\sqrt{-111})/(-2\sqrt{-111})$, kay sup & dah tung way $x = 4/37$ & $y = -18/37$; Q.E.D.

(20) GAI YUAN CHUAN TZU GWO SHIK. Dang (17) loon khaw A

---A--- chuan yi ping = $(M\ 0\ 1\ \infty) = (N\ \infty\ -2\ 4)$, hur
A=0 B=∞ $M = 6/(4-N)$ & $N = (4M-6)/M$ & hai ≠ whaw $M = N$
M N are $(2 \pm \sqrt{-2})$ neh U & V tsing; $U/V = s$, mo keen
1 −2 sup A. Π lo ≠ P = AM + BN, hur ken gai chu:
2 1 $M = (2Ax + 4y)/(2A - 2x + Ay)$ & $N = (2Bx+4y)/(2B-2x+By)$
3 2 & Π ping ≡ x fong $(-8A - 2AB - 12) + 2xy(2AB + A$
4 5/2 $+ B - 8) - y$ fong $(3AB - 8B + 8) + 4x(2AB + 3A + 3B)$
6 3 $-4y(3AB - 4B) - 12AB$, hur if $A = 0$ & $B = \infty$ (jak joy
−1 10
−2 7 $\Pi \equiv xy + 4y^2 + 6x + 8y = 0$ & $a=0$, $b=1/2$, $g=4$, $d=3$,
∞ 4 $h = 4$, $v = 0$ hur $\Delta = -24$, $e = -1/4$ & Π wot gwo.

(20 Hao) Way mik: yong y = Sx + c nai Π ping & yoo:

$x^2(S + 4S^2)$ & $x(c + 8Sc + 6 + 8S)$ shee chang pun ye kee hee, hur ← mik (6) y = −6 → ye mik ≡ (y = −6) tzu S=0, c= −6 & tzu S = −1/4, c = 4, mik ≡ x + 4y = 16. Hur kay sup way chung Pe ken y = −6, tzu q = (6) & x = 40 (mo tung peen). Lun way chung: yong twan sia fat & dah tung y + 6 = 0 & x −40 = 0. Shik, (4) x= −4, tzu y = 2 or −3. M= 1 & N = −2, kon chuan ← ℓ∞ → chirk, way AM + BN= P. Lun shik yong M=3, N= 2 & lun Π # shik x=−4 yoo M = 6, N = 3, joy. Nim A & B wing Π lo yen tung Λ. Joy y=2 (2) A & B poo tzu # gih Λ chuan, hur Λ & Π yoo see tung sup are (A B U V) nim U V joy shee P 1 2' x 1 ∞ tsing, mo fat 2' chan 3' meh. 4' Nim yen A=0 & B= K= A hur/ lun Π # = AA + BB = xk = woo −2 # on x. P Shik way maw p on P yong fat: (y −y')/(x − x') = 2 dy/dx =−(y'+6)/(x' + 8y' +8) hur maw p ≡ 2x + 5y = 2. If x =0, y = 2/5 & if y = 0, x = 1, hur mo kien wha maw, lyn P & 1' meh. Zai yong A & B jak lun Λ # & tung or mo tung Λ chuan 3 tzu meh.

(21) LUN LOON SHIK GWO & GAI TAN. Dang (18) yong chuan Λ ≡(M 1 2 ∞)/ = (4 N −4 −2 4), or

----Λ----			
M	N		& N = (−16 y +4 M)/(2 + M), hur tsing hai #
A=1	(H)	−4=B	U, V = (1 ± √−15)/ & ΠΛ # = A=1, B=−4 &
2	(V)	−2	ye #U, V, mo keen. 5 Chirk joy jaw shik
−2	(G)	∞=K	lun Π #, neh AM + BN = P on Π,
∞	(D)	4)	shik, A1 + B−4 = H tung loon ∞/ℓ &
6	(A)	1	A2 + B−2 = V poo tung ℓ; shik yoo Π # A,B,
−4	(B)	16	G,D H V C Way 6'' Π ping.
0	(C)	8	
4		0)	$M = \frac{2Ax + 4y}{2A - 2x + Ay}$ & $N = \frac{2Bx + 4y}{2B - 2x + By}$,

----

hur $\dfrac{(Bx + 2y)}{2B - 2x + By} = \dfrac{(-8A + 8x - 4Ay + 2Ax + 4y)}{(2A - 2x + Ay + Ax + 2y)}$, shee leet :

$x^2(AB + 4A - 2B + 16) + xy(-AB - 6A - 6B + 4) + 2y^2(2AB + A - 2B + 2) + 2x(-AB - 8A - 8B) + 4y(4AB + A - 2B) + 16(AB) = 0$; yong A =1 B = −4) (Nim khaw yong lun jak ye way # meh), hur shik:

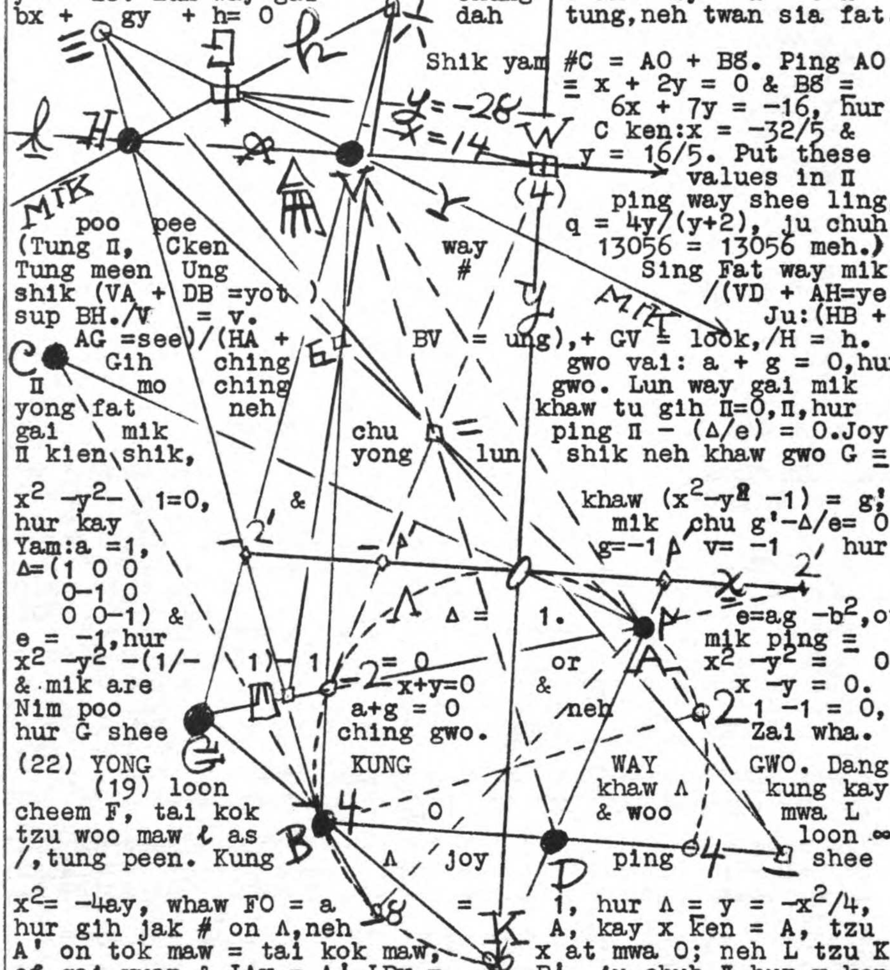
$\Pi \equiv 12x^2 + 13xy + 3y^2 + 28x - 14y - 32 = 0$, whaw hee shee:

a = 12, b = 13/2 g = 3 d= 14 h = −7 v = −32, hur e = −25/4

$$\& \ \Delta = \begin{pmatrix} a & b & d \\ b & g & h \\ d & h & v \end{pmatrix} = \begin{pmatrix} 12 & 13/2 & 14 \\ 13/2 & 3 & -7 \\ 14 & -7 & -32 \end{pmatrix} \neq 0,$$

hur Π wot gwo. Way mik yong y = Sx + c nai Π & zhui x^2 hee neh $(12 + 13S + 3S^2 = 0)$ & x $(13c + 6Sc + 28 - 14S = 0)$ hur $S = (\pm 5 - 13)/6$, or −4/3 & −3 & $c = (14S - 28)/(6S + 13)$;

(21 Hao) (1) if S = −4/3 c = −28/5 & h= 20x +15y +84 = 0
(2) if S = −3, c = 14 & mik v = 3x + y − 14 = 0. Nim mo
kien wha neh gih h (4x + 3y +28 = 0 if x = 0, y = −28/3 &
if y = 0, x = −7 & ju, gih v, if x = 0 y = 14, y=0, hur
x = 14/3 . Kay tung dah way chung Pe ken, neh x=14 &
y = −28. Lun way gai chung : ax + by + d = 0 &
bx + gy + h= 0 dah tung, neh twan sia fat.



Shik yam #C = AO + B8. Ping AO ≡ x + 2y = 0 & B8 = 6x + 7y = −16, hur C ken:x = −32/5 & y = 16/5. Put these values in Π ping way shee ling, q = 4y/(y+2), ju chuh. 13056 = 13056 meh.)

poo pee (Tung Π, Cken way # Tung meen Ung Sing Fat way mik; shik (VA + DB =yot) /(VD + AH=ye) sup BH./V = v. Ju:(HB + AG =see)/(HA + BV = ung),+ GV = look,/H = h.

Gih ching Π mo ching gwo vai: a + g = 0,hur gwo. Lun way gai mik yong fat neh khaw tu gih Π=0,Π,hur gai mik chu ping Π − (Δ/e) = 0.Joy Π kien shik, yong lun shik neh khaw gwo G ≡

$x^2 - y^2 - 1 = 0$, & khaw $(x^2 - y^2 - 1) = g$; hur kay mik chu $g' - \Delta/e = 0$. Yam:a =1, g=−1 v= −1 hur

$\Delta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ & Δ = 1. e=ag −b², or

e = −1, hur mik ping ≡ $x^2 - y^2 - (1/-1) - 1 = 0$ or $x^2 - y^2 = 0$

& mik are x+y=0 & x −y = 0.

Nim poo a+g = 0 neh 1 −1 = 0,

hur G shee ching gwo. Zai wha.

(22) YONG KUNG WAY GWO. Dang (19) loon khaw Λ kung kay cheem F, tai kok & woo mwa L tzu woo maw ℓ as loon ∞ /,tung peen. Kung Λ joy ping shee

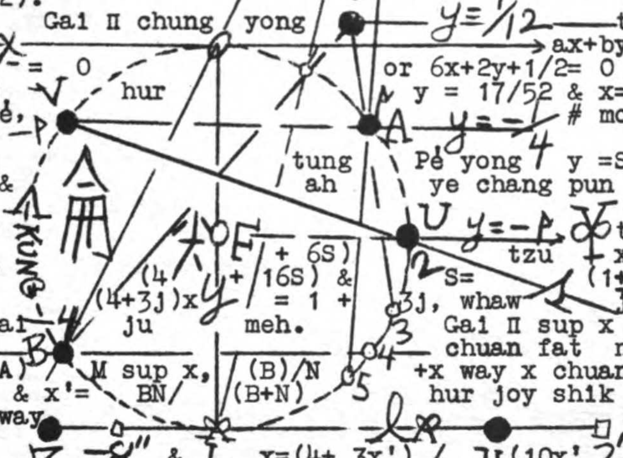
$x^2 = -4ay$, whaw FO = a = 1, hur Λ = y = $-x^2/4$,
hur gih jak # on Λ,neh A, kay x ken = A, tzu
A' on tok maw = tai kok maw, x at mwa O; neh L tzu K
of gai yuan & LAx = A',LBx = B', ju chuh & hur y ken
gih Λ # chu Λ ping ≡ y = $-A^2/4$, shik A = 1, x = 1, y = −1/
4; shik B; x = −4, y = −16/4 = −4; etc. Hur ping gih jak /
A/B ≡ $x(A^2 - B^2) + 4y(A - B) = AB(A - B)$, or kay leet:
x(A + B) + 4y(1) = AB. Way yeet yong A & B as ye jak
way # & M tzu N (chuan tan on Λ), neh AM sup BN = P of Π,
hur A/M = (A + M)x + 4y = AM & B/N = (B +N)x + 4y = BN.

(22 Hao) Hur $M = (Ax + 4y)/(A - x)$, $N = (Bx + 4y)/(B - x)$.
Yong jak chuan, A tzu B, with U=2 & V = −1 as hai #, hur yi ping = (M A U V) = (N B U V), or
(A −M)(B −U)(V −N) = (B −N)(A −U)(V −M) &if as joy, A = 1, B = −4, hur chuan yi ping = 5MN −7N = 10 −2m & M =(7N + 10)/(5N +2), N = (−2M + 10)/(5M −7) & kon chirk jaw.
Hai # whaw M = N, neh U chang V =(±3 +1)/2 or U = 2 & V = −1. Now it is easy to gai Π ping neh:

M	N
1= A	B =−4
2= U	U = 2
−1=V	V =−1
5 (G)	0
3 (D)	1/2

$(Ax + 4y)/(A - x) = (7Bx + 28y)/(B - x) + 10 : (5Bx + 20y)/\underline{B-x} + 2$;
or $x^2(-2A + 7B + 5AB - 10)$
$+xy(20A + 20B + 20) + 80y^2 + y(-28A + 8B)$
$+x(-5AB + 10A + 10B) - 10AB = 0$. Thus when A=1 & B = −4 (which are the two generative centers chosen for this example - they can be any two distinct #on Λ - zai lun shik), Π = $\underline{6x^2 + 4xy - 8y^2 + x + 6y - 4 = 0}$.
Nim ΠΛ see sup = A, B, U, V. Wan Π (yeet) thi? DAH.
Yen Π hee: a = 6, b = 2, g= −8, d = 1/2, h = 3, v = −4, hur Δ =
$\begin{pmatrix} a & b & d \\ b & g & h \\ d & h & v \end{pmatrix} = \begin{pmatrix} 6 & 2 & 1/2 \\ 2 & -8 & 3 \\ 1/2 & 3 & -4 \end{pmatrix}$
or discriminant Δ = 162 ≠ 0 &
$e = ag - b^2 = -48 - 4 = -52$, hur Π shee wot gwo;Q.E.D

Shik khaw Π # D, M = 3, N=1/2 hur (AM + BN = D) kay x = 2/3 & y = 1/12. Gih # G, M = 5, N = 0, & x = 1/2,y =1/2 neh way ping & dah tung gih 5A + OB, or x(1+5)+4y= 5 sup x(−4+0)+4y = 0, or 6x +4y −5 sup 4x −4y = 0 at #G (1/2, 1/2).



Gai Π chung yong twan ax+by+d=0 sia fat & bx+gy+h = 0 or 6x+2y+1/2= 0 & 2x −8y +3=0 hur y = 17/52 & x=−5/26 gih chung Pé, # mo shik.

Gai ye mik nai Π ping & O neh gih tung ah Pé yong y =Sx +c ye chang pun hee = (6 +4S−8S²) tzu x², (4c+16Sc+1 + 6S) x, hur c =−(1+6S)/ (4 + 16S) & S= (1±√(13))/ 4 or 8y + (4+3j)x = 1 + 3j, whaw J=√13 &. lun mik gai ju meh. Gai Π sup x yong chuan fat neh

ye but (A) M sup x, (B)/N +x way x chuan,whaw x= AM/(A+M) & x'= BN/(B+N) hur joy shik A(AUV) tzu B(BUV) way
$x' = \frac{(4-8x)}{(10x-3)}$ & x=(4+ 3x') / (10x' +8),
hur kay hai ## (± √37/5 −1)/4 = ye Πx # #. Ju way & gai ye Πℓ # joy,tzu Πk # tung gai yuan as Λ (joy Λ kung). Hur joy (A + A,U,V)" tzu (B + B,U,V)" or (1 + 1,2,−1)"tzu (−4 + −4,2,−1)" or (2 3 0 ℓ) = (−8 −2 −5 ℓ') & yi ping = ℓ(ℓ'+8) = (ℓ −2)(6ℓ' +30) or ℓ' = (−22ℓ +60)/(5ℓ −12) or ℓ = (12ℓ' + 60)/(5ℓ' +22) & hai # on ℓ = (±√13 −1)";Q.E.D

(23) YONG GWO SHEE GAI YEET. Dang joy loon khaw Λ gwo, kay chung = L & woo/ℓ = 2(U) −2; TTP wung = W = (−4) on y; tok maw x (mwa O = V), sup lwen ∞/ = Y. Khaw Λ ching gwo, hur Ls = siao pan kin = 1 & OL = tai pan kin = 1. Kay ping $\equiv x^2 - y^2 + 2y = 0$. Gih jak # A on Λ, $x = 4A/4 - A^2$ & $y = -2A^2/(4 - A^2)$, hur ping gih /AM

$\equiv A(4-M^2)(Ax + 2y) + 4AM(M - A) = 2M(x + y)(4 - A^2)$ & gih B/N $\equiv (4 - N^2)(Bx + 2y) + 4BN(N - B) = 2N(x+y)(4 - B^2)$.

Hur, if A = 1 & B = 4, tzu ping

AM $\equiv (4 - M^2)(x + 2y) + 4M(M - 1) = 6M(x + y)$ &

BN $\equiv (4 - N^2)\ 2(2x + y) + 16N(N - 4) = -24N(x+y)$.

AM sup BN = P # tung lo Π. Jak B chuan yi ping ≡

M = 2N/(3N −4) & N = 4M/(3M −2) ---Λ ----

Nim ΠΛ see # joy ≡ Π joy equation of use plicated for should be gai dah tung of tzu A/M U & V. The is too com- hence its # tung par using & B/N kon siang.

Λ			
1	M	4	N
1	A	4	B
2	U	U	2
0	V	V	0
−2	(P)	=A	1
4	(B)		1.6

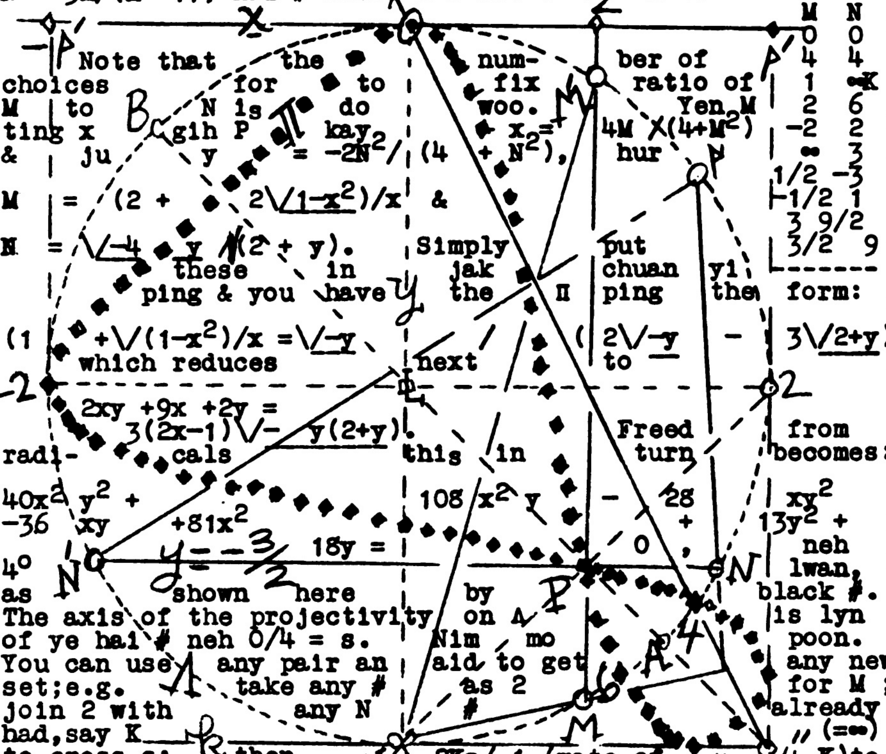
(24) YONG 𠃊 LOON GWO WAY SARM PUN LWAN. Way loon ching gwo ju siang way - yoo loon woo / ℓ tung lwen chung gih Λ as lwen yuan, hur gwo Λ loon chung at lwen woo on y. Gwo ping $\equiv x^2 - y^2 + 2y = 0$. Gih sin lwan, neh Π ($_2$=3°, or cubic), $x = 4M/4 - M^2$ & $y = -2N^2/4 - N^2$, hur P# on Π = WM sup NK' (whaw K' = woo # on x, or k). Khaw chuan yi ping: M = 2N/(3N−4) or N = 4M/(3N−2)

Hur Π $\equiv 16x^2 + 24x^2y - 12xy^2 + 5y^2 - 24xy + 8y = 0$. Some hook # joy shik Π lo. Yen $M = 2\sqrt{x^2+1}) - 2/x$ & $N = 2\sqrt{y})/\sqrt{y - 2}$ mo kien way Π ping neh $\sqrt{x^2+1}) - 1/x = \sqrt{y}/(2\sqrt{y} - 3)\sqrt{y-2}$, which leet yop siang. Jak E/ as M/N $\equiv 2(M + N)x + (4 + MN)y = 2MN$. Ye mik : $3y + 2 = 0$ & $4x - 2y = 1$. There are points symmetrically located but a cubic does not have a center as such. Kao zai way chang ho hook deem tung Π joy lo meh.

....Λ....	
M	N
0	0
2	2
2	1
4	1.6
3	12/7
−1	4/5
∞	4/3
−4	8/7
1.6	16/7

(25) POH HOH LWAN. Curves of the HOH seh have their points coordinated by projectively related points on, based on, the gai yeet, which can be yuan, tang, gwo, or parabola.

(25 Hao) The example in (24) is based on a hyperbola; joy vai shee lwen yuan Λ, called the POH HOH, or Peppermint. It has the x of M for x & the y of N for y, thus if ℓ be woo / & yℓ = W, with xℓ = R, then for any Π #, WM sup RN = P. Choose any # on Λ as M, then N will be fixed by jak chuan yi ping, shik joy (M 0 1 ∞) = (N 0 ∞ 3) or M = N/(N−3) & N = 3M/(M −1); hai # where M=N are 0 & 4.



M	N
0	0
4	4
1	∞
2	6
−2	2
∞	3
1/2	−3
−1/2	1
3	9/2
3/2	9

Note that the number of choices for M to fix ratio of M to N is do woo. Yen M ting x gih P kay $x_2 = 4M/(4+M^2)$, & ju $y = -2N^2/(4 + N^2)$, hur $M = (2 + 2\sqrt{1-x^2})/x$ & $N = \sqrt{-4y/(2 + y)}$. Simply put these in jak chuan yi ping & you have the Π form: $(1 + \sqrt{1-x^2})/x = \sqrt{-y}\,/\,(2\sqrt{-y} - 3\sqrt{2+y})$ which reduces next to $2xy + 9x + 2y = 3(2x-1)\sqrt{-y(2+y)}$. Freed from radicals this in turn becomes: $40x^2y^2 + 108x^2y - 28xy^2 + 13y^2 - 36xy + 81x^2 + 18y = 0$, 4° neh lwan, as shown here by black #. The axis of the projectivity on Λ is lyn poon. of ye hai # neh 0/4 = s. Nim mo any new You can use any pair an aid to get for M; set; e.g. take any # as 2 already join 2 with any N # had, say K (=∞) to cross s; then 2Ks/1 (mate of K) to sup Λ at 6. Now draw the ordinate through 2 & the abscissa through 6, to meet at Π # whose ken are x =1, y = −9/5, easily calculated from siang fat. #0 & 4 as Λ hai # are of course on Π. Two other intersections of Π & Λ are −2 & 6. Note the node marked P at 2K + 4"/−1'. Between P & −2, as well as kang −2 & 0 & 0 & P there are flexes, or # where curves changes from convex to concave & the tangent crosses. Gai shik yot mik = y = −27/20. P ken = x = 1/2, y = −3/2, or M (M'), N(N') = $\bar{4} \pm \sqrt{12}$ & $\pm \sqrt{12}$. 0/4 = 2(0+4)x + (4−0·4)y = 2·0·4, or 2x + y = 0. 4"/−1' or A/B joy = x + y +1 = 0; A, B = 2 + 2√2, whence for A, x= 1/√2, y = −1/√2 −1 & for B x = $\bar{1}$/−√2, y = 1/√2 −1 meh. Maw at −2 to Π is −2/−1', as can be yam yong −2 ken tung S way mu fun shee ling, hur slope of −2−1' is infinite; QED.

(25 Dai Ye Hao) HOH IS LOTOS, TREE OR LILY, KON POH HOH.
POH IS SLIM & THIN OR SLIGHT & SHIVE.
POH HOH IS PEPPERMINT, DOUBLY DESIGNED.

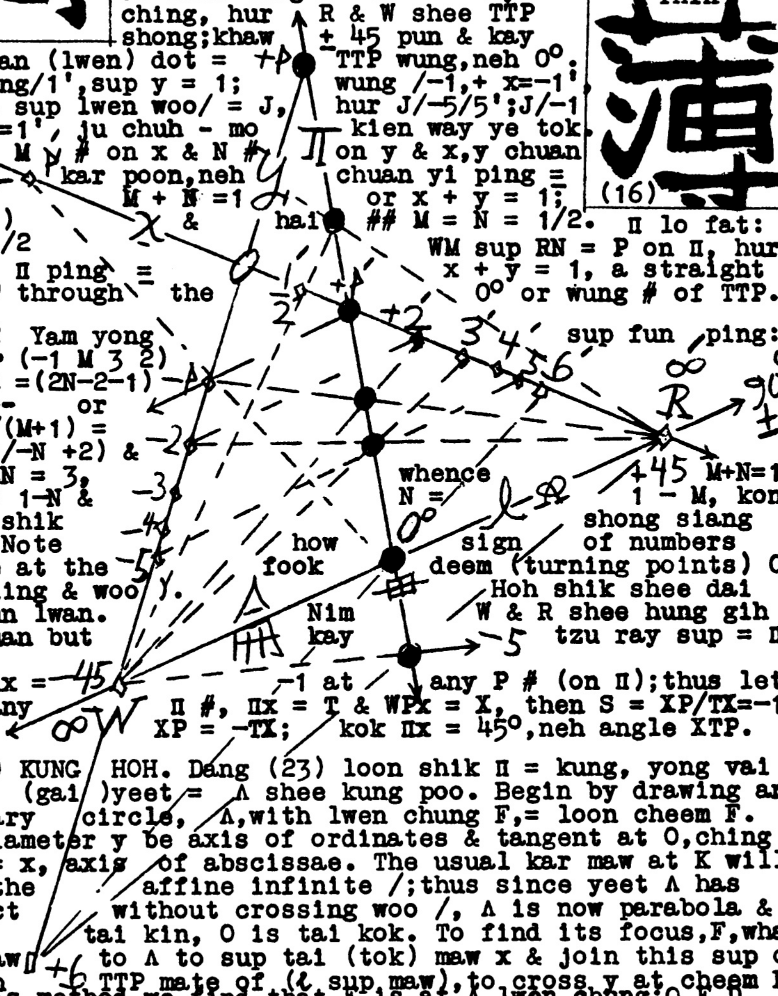
H O H
3942 - (140)
Lotos
荷
(8)

(26) CHAN HOH. Dang joy loon shik Π shee / (chih) a degenerate member of the Hoh family. Tsih wha ye jak / tung meen x & y, kay sup = 0, ken chu. x sup loon woo / ℓ, = R (∞') & yℓ = W (∞). Loon tok on x & y (neh harmonic scale), zo chee tung x. Khaw x & y ching, hur R & W shee TTP shong; khaw ± 45 pun & kay kang pan (lwen) dot = TTP wung, neh 0°. Hur wung/1', sup y = 1; wung /-1, + x=-1', khaw ℓ sup lwen woo/ = J, hur J/-5/5';J/-1 sup x =1', ju chuh - mo kien way ye tok. Khaw M # on x & N # on y & x,y chuan shee kar poon, neh chuan yi ping = M + N =1 or x + y = 1; & hai ## M = N = 1/2. Π lo fat:

P O H
9381 - (140)
Thin
薄
(16)

M (x)	N (y)
1/2'	1/2
0	1
1	0
2	-1
-1	2
∞	∞
W	R

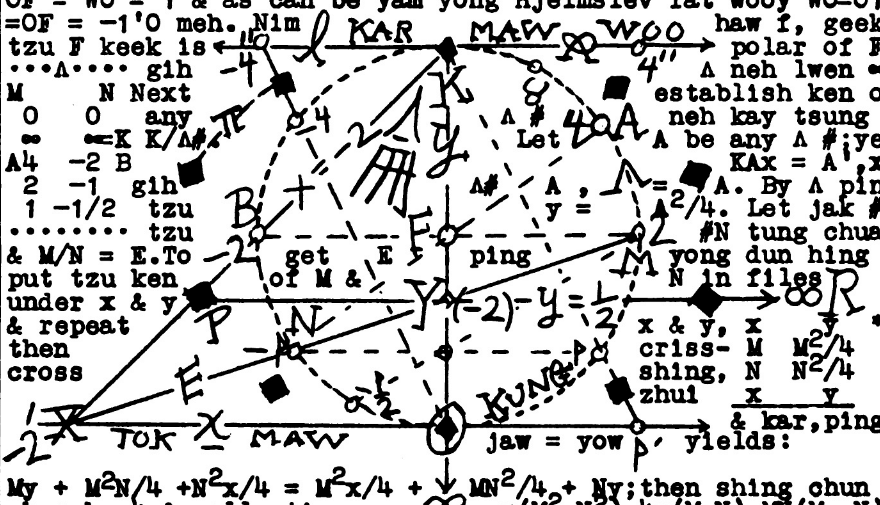
WM sup RN = P on Π, hur x + y = 1, a straight / Π ping = through the 0° or wung # of TTP. Yam yong sup fun ping: (-1 M 3 2) =(2N-2-1) or (M-2)/(M+1) = (-N-1)/-N +2) & 3M + 3N = 3, whence N = 1 - M, kon or M = 1-N & M+N=1, chirk shik shong siang jaw. Note how sign of numbers change at the fook deem (turning points) 0 & ∞ (ling & woo). Hoh shik shee dai yot pun lwan. Nim W & R shee hung gih ye chuan but kay tzu ray sup = Π.



S=dy/dx = -1 at any P # (on Π); thus let P be any Π #, Πx = T & WPx = X, then S = XP/TX=-1; or XP = -TX; kok Πx = 45°, neh angle XTP.

(27) KUNG HOH. Dang (23) loon shik Π = kung, yong vai (gai)yeet = Λ shee kung poo. Begin by drawing an ordinary circle, Λ, with lwen chung F,= loon cheem F. Let diameter y be axis of ordinates & tangent at 0, ching to y,= x, axis of abscissae. The usual kar maw at K will be ℓ, the affine infinite /; thus since yeet Λ has contact without crossing woo /, Λ is now parabola & y is tai kin, 0 is tai kok. To find its focus, F, wha jak maw to Λ to sup tai (tok) maw x & join this sup of x with TTP mate of (ℓ sup maw), to cross y at cheem F. By this method we find that F is at Λ lwen chung; Q.E.D.

(27 Hao) Now K is wung of TTP, the Ting Tang Poon, or fixed elliptic involution on the ideal / ℓ, & maw at 2 & −2, sup ℓ at 4" & −4", the + & − 45° #, thus e.g., any / on 4" will meet any / on −4" at right angles in the plane. Ju, R= (ℓ + lwen ∞ /) is mate of wung K, the O pun #. R is both + & − 90 just as K is ± 0°. These two pairs of mates fix the poon which is tang since its hai # are imaginary, neh the two # ω & ω', common to every possible circle, called the "circular points"; & any TTP shong subtend ching kok at jak meen deem. Now the same ye Λ maw at mwa 2 & −2, sup x at 1' & −1', ± unit # of the x tok which is both loon & lwen & can hur be laid-off yongkwooy; then K hop x # to Λ & O perspects Λ # to kar maw joy ℓ as hai zo chee, kwoo fat With O as vertex & OF = u = 1, let ken of O be x', y'= 0,0, then ping for kung in this vai is: $(x - x')^2 = 4u(y - y')$, or $(x-0)^2 = 4u(y-0)$, or $x^2 = 4y$, neh $y = x^2/4$. Let yℓ = W; nim OF = WO = 1 & as can be yam yong Hjelmslev fat wooy WO=O1' =OF = −1'O meh. Nim haw f, geek



tzu F keek is polar of F
•••Λ••••• gih Λ neh lwen ∞/
M N Next establish ken of
0 0 any neh kay tsung =
∞ ∞=K K/Λ# Λ# Let Λ be any Λ #; yen
A4 −2 B KAx = A', x
2 −1 gih Λ# A, Λ = Λ. By Λ ping
1 −1/2 tzu y = $A^2/4$. Let jak #M
•••••••• tzu #N tung chuan
& M/N = E. To get E ping yong dun hing *
put tzu ken of M & N in files
under x & y
& repeat x & y, x y *
then criss- M $M^2/4$
cross shing, N $N^2/4$
zhui x y
& kar, ping
jaw = yow yields:

$My + M^2N/4 + N^2x/4 = M^2x/4 + MN^2/4 + Ny$; then shing chun cher by 4 & collecting = $x(M^2-N^2)-4y(M-N)=MN(M-N)$; then fun chun by (M−N) & E ≡ $(M+N)x - 4y = MN$; Q.E.F.

Now the rule for this member of the Hoh family is: Ex = X, Ey = Y, & KX + RY = P, jak # on Π, which here is called an EPSILON CURVE, since its # are coordinated by E sup x & y. Hur X is got by yong y=0 nai E ping, neh: for P, x=MN/(M+N) & yong x=0 tung E ping way tzu P ken y = −MN/4. Next obtain M explicitly for the x ken, neh Mx + Nx = MN, or Mx − MN = −Nx, or M = −Nx/(x−N) & ki, M = Nx/(N −x). Ju for N gives 4y = −MN; N = 4y/−M. So far what we have holds good with every given projective relationship of M & N. Now we must particularise it by selecting three M # & their tzu three N #, kon chirk siang jaw. For simplicity & to have ye sup of Λ & Π a priori & integral let the double # be 0 & ∞ & lun shong be A=4 & B= −2, then cross-axis (sup cheh) is OK = y.

(27 Dai Ye Hao) Equation of corresponding cross-ratios: $(M\ 0\ \infty\ 4) = (N\ 0\ \infty\ -2)$, or $(0-M)(4-\infty)/(4-M)(0-\infty) = (0-N)(-2-\infty)/(-2-N)(0-\infty)$; since a simple algebraic sum involving ∞ & jak ho = ∞ & $\infty/\infty = 1$, the factors containing ∞ cancel out & we get: $-M/(4-M) = -N/(-2-N)$ or $\underline{M + 2N = 0}$ as jak chuan yi ping. Or $M = -2N$ & $N = -M/2$, which values put respectively in the explicit equations for M & N ,kon siang, give $M = -Mx/2 : -M/2 -x$, or $M = Mx/(M +2x)$, or

$M^2 + Mx = 0$, whence $\underline{M = -x}$. Ju, $N = 4y/-M$ or $N = 4y/2N$, or $N^2 = 2y$, whence $\underline{N = \pm\sqrt{2y}}$. Since M & N in this chuan always have opposite signs we take the negative root & fix $\underline{N = -\sqrt{2y}}$ for the radical form, then put these new explicit formsof M & N values in chuan yi ping & have instead of $(M + 2N = 0)$ now: $-x -2\sqrt{2y} = 0$ or $\Pi \equiv \underline{x + 2\sqrt{2y} = 0}$, which freed of radicals is: $\underline{x^2 - 8y = 0}$ the equation for the Epsilon curve exemplified here. Since in it $a = 1, h = -4$ & chun lun ting shee ling, $\Delta = -16$ & e $(ag=b^2) = 0$, hur Π wot kung & the locus of the black # is also a parabola; loon shik, with vertex at the origin & mwa K on woo / ℓ.

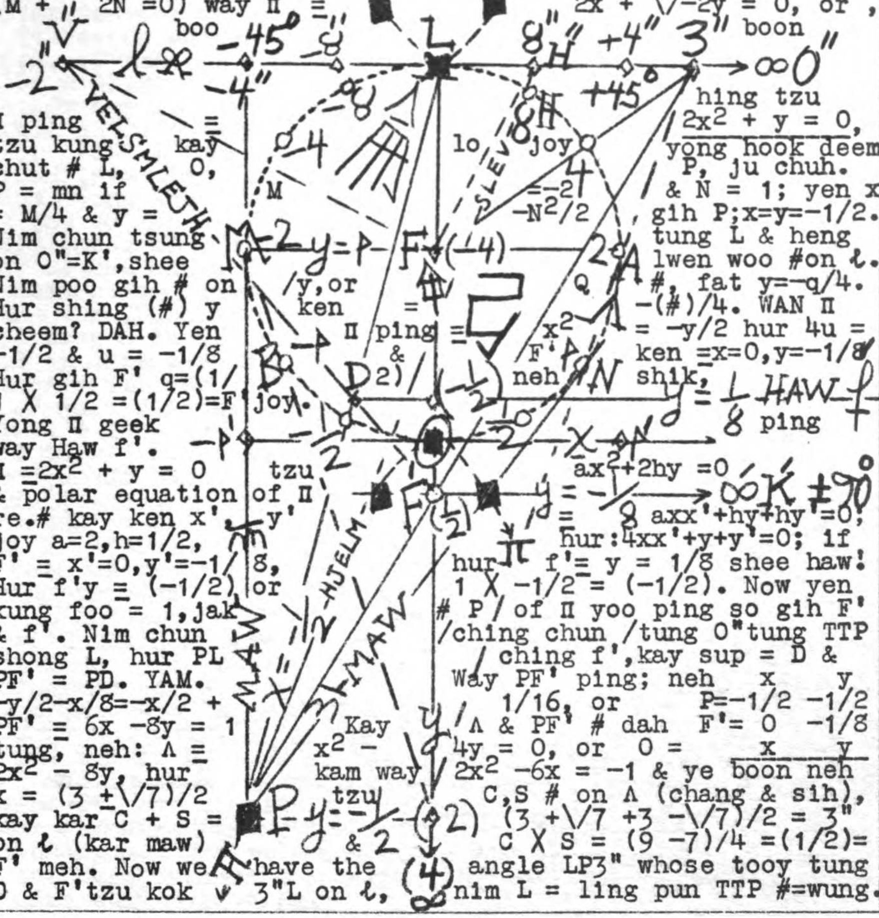
WAN Π cheem? In the equation for Λ neh $x^2 = 4uy$, where $u=1$, the so kang vertex O & cheem F, neh u,= so kang O & haw,or pan par,= 2fF,whence par = 4u. Thus nai ping Π, $4u = 8$ & $u = 2$, hur,yen u yang,,if F' be Π cheem, OF' = 2 & $x=0, y=2$ are ken of Π focus. To find this # on y note that the Q # or M X N of any E/ through the # $y=2$ will solve the problem. Thus, yen $y = 2$, E joy $\equiv x(M+N)-8 = MN$; but $x=0$,hence $MN = -8$, let $M = 4$,then $N = -2$ & E/ whaw $M=4$ & $N=-2$, sup y at (-8) whaw $y = 2$. Or,in general, to state Q in terms of y (as given on $x=0$,the y/);if the y/ is identical with the E/, $M = -N$ & fon yee; whence $y \equiv x(M -M) -4y = -M^2 = Q$ or $-4y = Q$, or $y = -Q/4$; shik $y = 2$, $Q = -8$ & the wan y # is (-8) the product of any two Λ # such as 4×-2, 2×-4 & the heng itself will be $+\sqrt{8} \times -\sqrt{8}$. Zai lun shik ju chuh

Geometrically:given M on Λ,use OK as axis,then if A & B be jak chuan shong, fat: MBy/A,+Λ = N; M/N = E,+ x = X,+y,=Y, & KX + RY = P on Π; Q.E.D.

Khaw lun shik chuan,neh $(M\ 0\ \infty\ 1{=}A) = (N\ 0\ \infty\ 2{=}B)$, then yi ping is $2M - N = 0$, or $M = N/2$ & $N = 2M$, whence $M = \underline{2Mx}$ & $N = 8y/-N$, hur, $M = 3x/2$ & $N = \sqrt{-8y}$, whence Π $\overline{2M - x}$ ping $\equiv 3x = \sqrt{-8y}$ or freed of radicals, $9x^2 + 8y = 0 \equiv \Pi$. Shik: if $M = 1$, $N = 2$ & with $x = MN/M{+}N$ & $y = -MN/4$, shik $x = 2/3$ $y = -1/2$ ken gih P. Put these values in Π ping & yoo: $2 = \underline{\sqrt{-8} \times -1/2}$ or $9(2/3)^2 + 8(-1/2) = 0$; Q.E.D.

(28) MAW HOH. Dang (25) loon yong Λ vai yeet kung kay ping $\equiv x^2 = 4y$, ju booy fo. Joy lwan Π fat $\equiv m + n = P$, neh the tangent m, mwa M, meets maw n,contact # N, at #P on lo Π, with M & N projectively related on fundamental conic Λ. With O (0) & K (∞) as hai ##, jak A = 2 & B = -1, as the non-united pair to fix the projectivity on Λ, then cross-

(28 Hao) ratio (MOKA) = (NOKB),neh −M/(A−M) = −N/(B−N) or
AN = BM, whence when A=2 & B=−1; M = −2N & N = −M/2.Chuan.
Chuan sup cheh is ye hai # lyn neh OL,joy, = /y. ···Λ···
Hur way sin shong fat: MBy / A,+ Λ = N;shik jak M (M N
as −2,then −2 X −1 = (2):2 = 1 = N. Tzu toy: yong (O O
x y dun hing way −1 + x = −x/2 + 2y as / (L∞ = ∞K
(2)O −1/2 (2)A, or 3x −4y = 2; dah tung with Λ (A 2 −1 B
A 2 1 ping, x^2 −4y = 0, gives(3 ± 1)/2 = 2 (−2 1
x y for A & 1 for N; Q.E.D. Chien gai Π. ··· ···
Yen ping gih ye jak # lyn of Λ is x(M+N)−4y =MN,
if ye # tung as maw,shik: m = x(2M) −4y =M^2 & ju,lun maw
n = 2Nx −4y = N^2. m − n = 2x(M−N) =M^2−N^2, or x =(M + N)/2.
Nai m = 2M(M+N)/2 −4y = M^2, hur, y = MN/4. Yong yi ping ho
hur if M = 2x −N, poo M = 2x + M/2, or M = 4x &, yen
N = 4y/M, hur N=4y/−2N or N = √−2y. Yong nai yi ping
(M + 2N =0) way Π = 2x + √−2y = 0, or ,
boo boon



Π ping
tzu kung kay
chut # L, O,
P = mn if
= M/4 & y =
Nim chun tsung
on O"=K',shee
Nim poo gih # on
Hur shing (#) y
cheem? DAH. Yen
−1/2 & u = −1/8
Hur gih F' q=(1/
1 X 1/2 =(1/2)=F' joy.
Yong Π geek
way Haw f'.
Π =$2x^2$ + y = 0
& polar equation of Π
re.# kay ken x', y'
joy a=2,h=1/2,
F' = x'=0,y'=−1/ 8,
Hur f'y = (−1/2) or
kung foo = 1,jak
& f'. Nim chun
shong L, hur PL
PF' = PD. YAM.
−y/2−x/8=−x/2 +
PF' = 6x −8y = 1
tung, neh: Λ =
$2x^2$ − 8y, hur
x = (3 ±√7)/2
kay kar C + S =
on ℓ (kar maw)

hing tzu
$2x^2$ + y = 0,
yong hook deem
P, ju chuh.
& N = 1; yen x
gih P;x=y=−1/2.
tung L & heng
lwen woo #on ℓ.
#, fat y=−q/4.
−(#)/4. WAN Π
= −y/2 hur 4u =
ken =x=0,y=−1/8
shik,
ping
axx'+hy+hy'=0;
hur:4xx'+y+y'=0; if
hur f'= y = 1/8 shee haw!
1 X −1/2 = (−1/2). Now yen
P / of Π yoo ping so gih F'
/ching chun /tung O"tung TTP
/ ching f',kay sup = D &
Way PF' ping; neh x y
1/16, or P=−1/2 −1/2
Λ & PF' # dah F'= 0 −1/8
4y = 0, or O = x y
$2x^2$ −6x = −1 & ye boon neh
C,S # on Λ (chang & sih),
(3 +√7 +3 −√7)/2 = 3"
C X S = (9 −7)/4 =(1/2)=

F' meh. Now we have the angle LP3" whose tooy tung
D & F'tzu kok 3"L on ℓ, nim L = ling pun TTP #=wung.

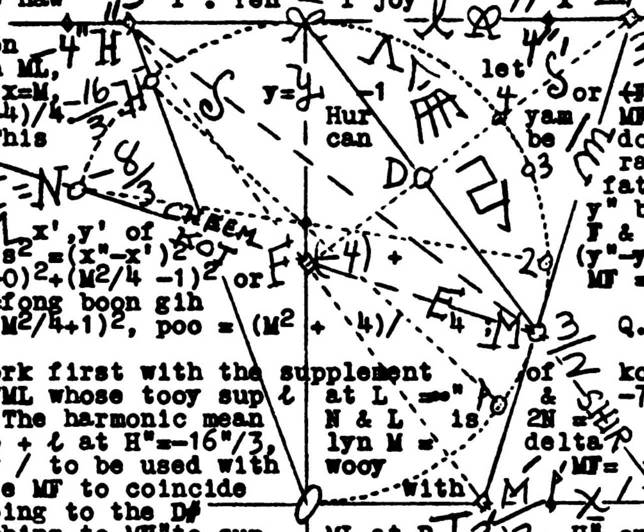
(28 Dai ye hao) If PD = PF', which is to be proved, then kok F'DP = kok PF'D & F'D is perpendicular to the bisector(pan daw) of kok DPF'. This pan daw is the so-called Hjelmslev line used in loon wooy, or rotation of segment PF' on vertex P to coincide with leg PD. The geometry of this work has been explained often to you (kon do lun fo nai tai & siao shu), now we will give you also the algebra, which is quite easy when the gai yeet is kung Λ as here. First organise the TTP on ℓ; this is an elliptic involution whose mated # are cut on ℓ by corresponding rays of the circular poon pencil whose hung is F; then any two hai zo # on ℓ are shong in TTP if their lyn with F subtend a right angle at F, which is both lwen & loon. The ± 45°# are 4" & −4"; 04ℓ= 4" & 0−4ℓ = −4"; or (4)2ℓ & (4)−2ℓ are same. The mate of infinite # on ℓ, neh 0", is L, the TTP wung (linear center) pairs divide each other hence poon tang. Thus ℓ # can be shown by doubly primed numbers as well as by degrees, which makesfor easier calculation. We have the three necessary pairs to fix the chuan which is poon, hur yoo hai tzu, &any pair can be reversed; thus sup fun = (∞ 0 4 ℓ) = (0 ∞ −4ℓ') (Note that L is ∞ & 0" is 0, opposite of vai tung x where K' is ∞, the very same lwen woo # as 0", viz, ± 90°of TTP.) Hur on ℓ: (ℓ −4)/−4 = (ℓ' +4)/ℓ', or $\ell\ell$' −4ℓ' = −4ℓ' −16 & $\ell\ell$' = −16, whence if ℓ be any ℓ # & ℓ' its TTP mate, their arithmetic product is always −16, when they are shown as sums, or hai zo #, neh sum of the two # on Λ cut by any same / through the ideal # on ℓ. Neh the ℓ # is chong gih kar poon & where all lyn of tzu # concur, whose sum is the same. Hai # of such kar poon are L & the arithmetic mean (swan kang) which is same for any two corresponding # of the involution, neh ℓ/2 which is the lun sup of Λ geek tzu ℓ, or the two tangents from ℓ to Λ make contact at L (= K) & ℓ/2; shik for ℓ = 3", hai # are L & 3/2. The TTP mate of 3", = −16/3"; thus −16"/3 & 3" subtend ching tok tung F.

Next find the formula for bisection. One # on ℓ will serve when joined to the vertex of any angle to bisect it for the angle subtended at any plane # by lines through a given pair of ℓ #; thus kok LP3" = kok LF3" = kok LM3"etc. (Loon liang, of course!) H" is the # kang 3" & L we must get & FH" is pan daw for kok LF3"; PH" for kok LP3", & ju. H" is not the lwen mid # of 3"L. Here is the way to find it. F3"Λ = C = 4. Thus arc 4L tzu 3"L, or to make it quite fat, let C & C' on Λ tzu kok twan on ℓ, then pan dot on Λ gih arc CC' is got by shun lung or nee gai, neh:(OCHC')=−1 or C(C' −H)/C'(C −H) = −1 & H = 2CC'/(C+C'); thus when C'= L , joy, C'= ∞, H = (2 X C X ∞)/(C + ∞), or H, the harmonic mean between C & C'(shun kang), = 2C when C' = ∞ = L, joy. C = 4, shik, hur H = 8. Ki, OHℓ = H"= 8" & FH" = F8" is pan daw of kok LF4 = LF3", & PH" = P8" is pan daw of kok PL3" neh dah Hjelmslev /. Next, we must draw the base of the isosceles Δ, neh / F'D , ching to PH". Simply get TTP mate of H", neh V" = −16/H" = −16/8" = −2" & lyn with F', the Velsmlejh / which wooy PF' = PD, for V"F' + f' = D(x=−1/2, y=1/8. Fat:FℓΛ=C $=\left(\ell \pm \frac{\sqrt{\ell^2+16}}{72}\right)$; yong C = + boon; mo kien yam.

Zai do lun shik meh.

(29) HJELMSLEV FAT TOY. Yong kung A kay ping = $x^2 = 4y$. Jak #M (3/2) on A kay ken: $x = M$, $y = M^2/4$ & tzu maw m = MM = $2Mx - 4y = M^2$. Khaw mx = T, hur way T ken yong y = 0 & x = M/2 (shik 3'/4). my = Q, way kay ken yong x = 0 nai m ping & yoo −M fong / 4, neh Q = M fong & tzu y = −Q/4, shik (9/4)= Q & tzu y = −9/16 joy. ml = 2M" = 3" shik. A cheem = F = (−4) kay ken, x = 0, y = 1. Haw f = lwen woo / neh y = −1, geek tzu keek F. Kung foo (1) way MF = Mf, hur yong loon wooy tung meen way yam & tzu toy gai shik.

Draw MF to cut A lun at N, = −4/M (−8/3) & to meet ℓ / at (M + N)" or (M −4/M)" or (M^2-4)"/M (shik −7"/6). We want finally to prove that Mf = MF, i.e. that for any #M on parabola A distance to focus F = that to haw f'. Yen f joy ℓ // x the ching so to f liang on shir of M, neh ML, MLf=D' & D' ken: x=M, let S or (F)/4, neh MF = MD'. Hur can yam be done by radical fat thus:



This if x", y" be ken of M, x', y' of F & s = so MF then $s^2 = (x"-x')^2 + (y"-y')^2$, hur $MF^2 = (M-0)^2 + (M^2/4 - 1)^2$ or MF = $(M^2+4)/4$. Ju, MD' = fong boon gih $(M-M)^2 + (M^2/4+1)^2$, poo = $(M^2 + 4)/4$ Q.E.D.

Let us work first with the supplement of kok D'MF neh kok FML whose tooy sup ℓ at L = ∞" & −7"/6, /F + A = N. The harmonic mean N & L is 2N = H= −16/3 Wha OH to + ℓ at H"= −16"/3, lyn M = delta the Hjelmslev / to be used with wooy / MF = MD, neh rotate MF to coincide with ML the F# going to the D# by wha FV" ching to MH" to sup ML at D. H" on ℓ is the Hjelmslev # & V" its TTP mate is the Velsmlejh point. Kok H"FV" is right. By formula of booy fo neh ul' = −16 we find V" = 2M, for H = 2N = −8/M, then −16/ (−8"/M) = 2M"= V" meh. Now we can get the D'# by shun lung, neh (LDMD') = −1, neh yen L = ∞#, DM = MD'. Using the fat siang: MD' fong = DM fong. Or, now applying Hjelmslev fat directly: yen supplementary kok pan daw are always perpendicular, maw m is bisector of kok D'MF & hur V" is Hjelmslev # for this angle & tzu shong H" is Velsmlejh #, & H"/F will cut ML at D' to way FM = MD'; Q.E.D. Thus H"F is lwen // (mo loon // LMD', hence they meet at lwen ∞ # D'. MD' = 25/16. In TTP joy mates are: ℓ tzu ℓ', L tzu 0" or ∞" tzu 0", 4#" tzu −4", the 45 pun #), & ye tsing hai # shee ω tzu ω & ω' tzu ω', hur ω, ω' = $\sqrt{-16} = \pm 4\sqrt{-1}$", ye "yuan"## on woo/ ℓ.

(30) GAI FAT. Yong A kung $\equiv x^2 = 4y$; wha E/ on F (–4) kay A sup = A (har) & B (siang) tzu FxA & FℓA. WAN A & B # & tzu yam? DAH. Kon booy fo. Mun # on ℓ as ℓ & on x,"x, hur tung E / Ex = x, Eℓ = ℓ & 1x (shing) = (–4); x = –4/ℓ & ℓ = –4/x. The A & B # will be on the same E / through F shik the E/ which cuts x at 4'/3 & ℓ at –3" has A = 1,B = –4; lun, if E/ sup x at –4'/3 & ℓ at 3",it has A = –1, B= +4. In calculating the value of B we must use the plus root , neh B = $\ell + \sqrt{\ell^2 + 16}$ / 2 whenever B is on the yow between L & O reading deosil; if B is on jaw ,neh LO be widdershins, use the negative root & B = $\ell - \sqrt{\ell^2+16}$) /2. But with A, given in terms of x, both sides use the plus root, neh always A = $2\sqrt{x^2 + 1}$ –2)/x. .

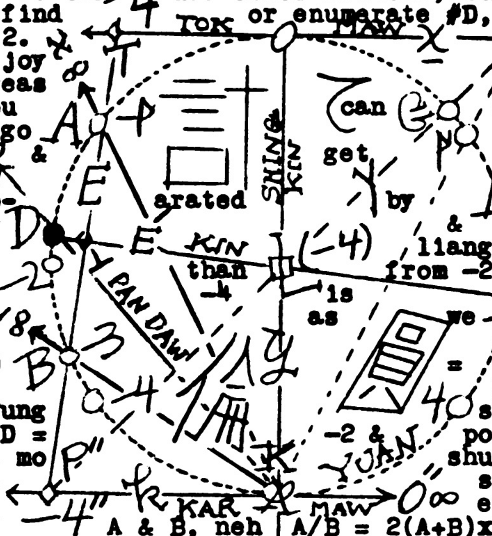
Shik: yong ∞ on ℓ tzu $\sqrt{-4}$ on x, then A=$\sqrt{-3}$ –1)/$\sqrt{-1}$ & B = ($\sqrt{-1}$ –$\sqrt{3}$), whence AB = –4. Or let ℓ = ∞',neh kam or –$\sqrt{-4}$ & x the same, gives 1x = –4 & tzu A=$\sqrt{-3}$ –1/–$\sqrt{-1}$ & B = –$\sqrt{-1}$ +$\sqrt{3}$ & AB = –4, as before.

Fon yee: $\ell = (B^2 - 4)/B$ & $x = 4A/(4 - A^2)$; note that these values of x pertain to # on x /, not to # on A.Thus the sum of A + B = ℓ, but the sum of $\ell + x = \ell^2 - 4/\ell$, or $\ell + x = \underline{(A+B)^2 - 4}\,/(A+B)$; ju chuh. It is good practice to work these rules backwards & forwards,especially those involving radicals, to establish the formulae which will be useful & save much time & work in the other calculations. Here we show the connection between the involution with F as chong & in one case tzu # on ℓ & x/ & in the other with tzu tan on kung A. The / of F with ∞ & ∞' on ℓ will be the two double of self-perpendicular maw from yuan // # to the focus.

(31) GAI tung chung,tok K, ju chuh = 2. OK = y, neh axis of abscissae. KUNG peen maw ju tsung xℓ= PAN (29) x ,mwa kwoo; (axis K', DOT. Dang lwen = gai yuan kay(–4) O, kar maw k, mwa pan = LO = 1, kin of ordinates), x heng, kℓ = 0"; nim K' = 0",∞ #.

WAN JOY GAI PAN DOT TUNG YUAN TWAN. Problem here is to calculate the mid point of a circular arc, such as AB, to have the number, as D,tzu D' on scale tangent x, jst as, shik, A = –1 & lun kwing = B = –3 Using a circle, simple nee gai does not dah wan as with a parabola,kon (29) Geometrically we can have no trouble at all.Jak kok BKA tung meen; count its legs wid as usual neh let BK = beta, to sup ∞/ℓ at beth; KA = alpha, to + ℓ at aleph, & KD = delta to + ℓ at daleth & A at D,such that arc AD = DB on A;D is pan dot of AB & kok DKA (alpha delta) = kok BKD (delta beta) & either of these,half kok of whole angle BKA (alpha beta; in short delta is pan daw or bisector of kok BKA,or if we were to rotate a length on alpha to an equal so on beta, delta would be the Hjelmslev /,used in the operation to which the Velamlejh / would be perpendicular;kon booy.

(31 Hao) Tsih wha ye jak / as alpha & beta to sup at K & then wha yuan A through K & establish the rest of the gai yeet procedure. For convenience we have assumed that alpha cuts x at -1' & A at 1, while beta sup A at -3 & x at -3', to make it definite & (mo fat chan) integral & easier! Let /AB = E,to + x at -3/4' & k at -4", neh A+B = p & AB = q & Ex = T = q/p, neh AB/(A+B). Having drawn the circle with unit radius the # A & B will correspond to their primes on x. The angle alpha beta can be bisected internally by delta through vertex K & #D on A will be at the sup of E' which is through L & ching E. Otherwise we can use compasses;lay off equal so on alpha & beta as radii with chung K, then from these ends as / sin chung ,ping pan kin sup at # on which to lyn K for delta; the usual method of bisectin g an angle. We can use other methods, but the problem here is to find or enumerate #D, which is not #-2, nor 3/2.



D is the actual joy lwen mid #of arc AB,whereas the scale on A mo lwen. You can see this, for as you go from O round the yuan & get nearer to K, the inter- vals between numbers sep- arated by equal spaces get smaller & smaller.Thus by lwen liang from -1 to -2 is more than from -2 to -3 & be- tween -3 & -4 is still less & so on as we approach ∞A. Yong kwooy & see that AD = DB Tung shun (0 - 1 D -3) = -1,way D = -3/2. Tung shun (-1 D -3 ∞) = -1 makes D = -2 & poo,(∞-3D-1) =-1. Hur dah fat mo shun lung. Here is the correct solution. Write equation of E / in terms of A & B, neh /AB = 2(A+B)x +(4-AB)y= 2AB, shik 2(-1 + -3)x +(4 - -1•-3)y = 2•-1•-3, or -8x+y= 6. We will do the whole thing first in algebra to make it perfectly general so as to apply to any A interval Thus E = always 2(A+B) for hee (coefficient) of x & of y shee (4 -AB) & ting (constant) is 2AB,which we do not need here. Differentiate E & have -2(A+B)/(4-AB),which is slope of E/ in terms of a & b the coefficients of x & y,& eliminating x & y. When two / are mutually ching their respective slopes are negative reciprocals of each other;if -a/b be slope of E, then sia of any / perpendicular to E, shik, E' joy, has dy/dx = b/a; hur lyn of D & L, neh E'has for its slope = (4 -AB)/2(A+B). Now, yen E' on L, (-4) is product of D X lun kiwng, neh -4/D. Put D & -4/D as the ## & have equation LD = 2(D -4/D)x +(4 - D•-4/D)y = 2•D•-4/D, or LD =(D^2 -4)x + 4Dy +4D = 0,whose dy/dx =(4 -D^2)/4D,hur: (4 -D^2)/4D = (4 -AB)/2(A+B) or D^2(A+B)+2D(4-AB)=4(A+B).

(31 Dai ye hao) Now we have pan dot D in terms of A & B & it is a simple matter to make it explicit, thus:

$$D = \pm \sqrt{4(A + B)^2 + (4 - AB)^2} - (4 - AB) \;/\; (A + B).$$

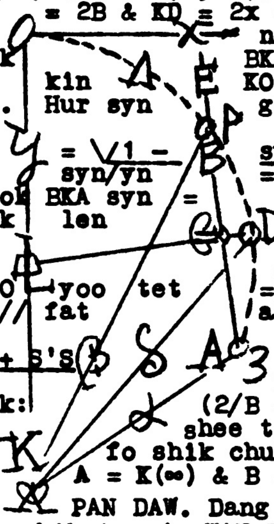
Shik: if A = −1, B = −3, then AB = 3, A+B = −4 & D becomes $(1 \pm \sqrt{65})/4$. The plus or minus root will apply to the internal or external pan dot of the angle according to placement of A & B. Lun shik, suppose A = 0 & B = 2, then pan # will be $\pm 2\sqrt{2} - 2$, kon G tung dang booy. Suppose A = 0, B= ∞, then

$$D = \pm\sqrt{4\infty^2 + 16}) - 4 \;/\; \infty, \text{ or } \pm 2\infty/\infty, \text{ neh } \pm 2. \text{ If}$$

A = 2, B = −2, then $D = \pm\sqrt{64} - 8/\ 0$ or $\pm 8 - 8)/0$ or, 0 & ∞.

Using A chung L as hung of circular poon to sup x, we find that mates on x multiplied together = −1; neh xx' = −1 is yi ping, neh (0 1 −1 x) = (∞ −1 1 x'). Another useful rule concerns the value of LxA. Let gamma be any kin tung L to sup x at ,e.g.,1' & A at G (kang 1' & L), then, the plus root neh $+\sqrt{1' + 1} - 2/\ 1' = G$, i.e. G joy = $2\sqrt{2} - 2$. Let the x # = 4/3, then $2\sqrt{x^2 + 1} - 2 \;/\; x = 1$ & this is fat. What would the kam boon signify?

(32) KOK SWAN FAT & GAI. Dang joy lwen shik ye jak # A & B (A chang B) on gai yuan A (A = 3, B = 1), hur KD (delta) = pan daw gih kok BKA, or alpha beta (KA = alpha, KB = beta), & D = $(\sqrt{65} - 1)/4$, (len 1.765··). Ping gih KA ≡ 2x−Ay =2A; gih KB ≡ 2x −By = 2B & KD ≡ 2x −Dy = 2D (ju chuh). Slope of alpha = 2/A



neh dy/dx gih alpha ping; etc. Syn gih kok BKA = AB /2, whaw AB = so A/B (kot) & 2 = kin KO. AB = 4(A −B)/(4+A²)(4+B²) neh shik 8/√65. Hur syn gih kok BKA = 2(A −B) / √(4+A²)(4 +B²) or 4/√65. Yn BKA = √1 − syn² or √(4 −AB²)/2 neh 7/√65. Tet = syn/yn = AB /√(4 − AB²) or 4/7. Shik: joy gih kok BKA syn = len .496; yn = .868 & tet = .57142 tzu kok len 29°45' meh. Lun gai way tet = 2(A −B) / (4 + A·B) (fat). Nim kok kang ye /(alpha ≡ ax +by +g = 0 & (beta ≡ a'x +b'y +g' =0 yoo tet = (ab' −a'b)/(aa' −b'b); hur if ye tooy // fat ab' −a'b = 0 & if ching, aa' + bb' = 0. Yong sia tet gih kang kok = (S' −S)/1+ S'S whaw S sia gih alpha &S' gih beta, count from alpha to beta wid. Gih kok BKA shik: (2/B −2/A)/(1 +2·2/B·A) = 2(A−B)/(4 +A·B) shee tet, Q.E.D. (Tai Shu KHO NGU XXV, 6). As fo shik chuh fat whaw A = A jak # & B = 0; lun if A = K(∞) & B shee lun jak A #; zai meh

(33) LUN YAM PAN DAW. Dang (31) lwen yong gai yuan A tung jak # A, shik A = 1. WAN D shee lwen pan dot gih arc OA, neh KD = pan daw gih kok (g'y) = OKA. MEEN DAH. FAT: syn 2D = 2·syn D · yn D, whaw D tzu kok D = OKD, etc.

(33 Hao) OD = z, OA = z'; KD = g, KA = g'Kon MAR SIANG(14)
& ZHI GAH (7) whaw g tzu j. Nim syn D = $z/2g = D/\sqrt{4+D^2}$ &
2·syn D = $2D/\sqrt{4+D^2}$. Yn D = $g/2 = 2/\sqrt{4+D^2}$, hur syn 2D =
syn A = $4D/(4+D^2)$ = x for D = z'/2 for A; thus wooy OA to
x/, bisect it & have x for D. From x# for D wha // y,+A,=D
Lwen yong kwooy, loon yong Hjelmslev fat.

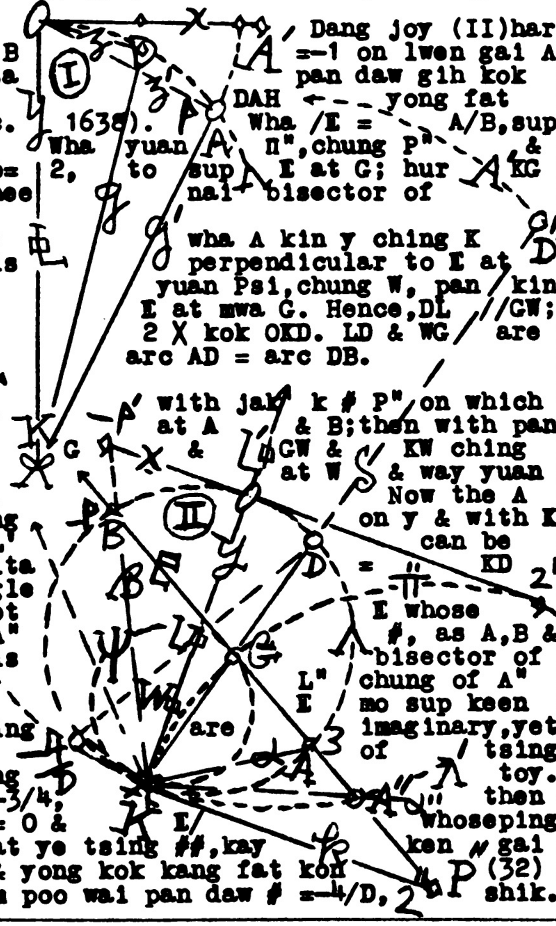
Further to prove value of D as given in (31,kon p.30)
yong tet fat, neh

tet A = tet 2D = 2 tet D /(1 − tet^2 D).

tet D = z/g = D/2, hur 2 tet D = D & tet^2D = $D^2/4$, hur

tet A = tet 2D = $4D/(4-D^2)$ = A/2, hur $4A - AD^2 = 8D$ & ki

D = $(\pm 2\sqrt{(4+A^2)} - 4)/A$. Yong (31)fat whaw A = 1, B = 0;yam!



(34) LUN PAN DAW FAT. / Dang joy (II)har
khaw jak # A=3 & B =-1 on lwen gai A
yuan & WAN (KGDD")delta pan daw gih kok
BKA, neh alpha beta? DAH ← yong fat
chu GALILEO GALILEI (c. 1638). Wha /E = A/B,sup
kar maw k at P" = 2". Wha yuan A n",chung P" &
pan kin = P"K, neh 4/p= 2, to sup E at G; hur KG
= dah pan daw delta shee nai bisector of
angle BKA; Q.E.F.
Nim: is wha A kin y ching K
to sup GW at W.GW perpendicular to E at
G, then GW = KW & yuan Psi,chung W, pan kin
= WG = WK, maw E at mwa G. Hence,DL //GW;
kok OLD = LWG = 2 X kok OKD. LD & WG are
both ching E & arc AD = arc DB.

Suppose we tsih with jak k # P" on which
draw jak E /,+ A at A & B;then with pan
kin P"K wha n"+ A at GW & KW ching
E & k at G & K meet at W & way yuan
psi to contact E at G. Now the A
yuan vai: kay chung on y & with K
mwa, otherwise L or L" can be
any # of y; yet delta = KD
will always bisect angle 2"
subtended at K by kot E whose
kwing are ye EA or EA" #, as A,B &
joy A",B"; shik KD is bisector of
kok B"KA", etc. Khaw L" chung of A"
jak below W such that E mo sup keen
A", then the A & B kwing are imaginary,yet
same delta is pan daw of tsing
kok, as can be yam yong toy.
Let L" ken be x=0,y= −3/4, then
A" = $2x^2 + 2y^2 + 3y + 1 = 0$ & whoseping
= 4x +7y +6, cuts A" at ye tsing ##,kay ken gai
& kay lyn with K gai & yong kok kang fat kon (32)
ju chuh yam delta. Nim poo wai pan daw # = −4/D, shik.

(35) LOON TZU. Tsih lwen kung Λ, kin y + loon woo / ℓ = TTP wung ,ling pun . Λ maw mwa 2/mwa −2,+ y =loon chung L gih Λ loon yuan. yΛ = 0 on x & K on lwen woo /= k, kar maw,/ 2,−2,+ ℓ = +,−,45°#. L/+45,+ x,Λ = 1', 1. K− 1 = beta,+ ℓ = beth.K−3 = alpha,+ ℓ,= aleph. Shik A = 3,B = −1;A/B= E way kok BKA; WAN kay pan daw? DAH.

Lwen liang 0°/45 = 0°/U,= hung of yuan poon but, kay shong sup ℓ at TTP shong, way ching kok at U.AB = E,+ ℓ = N, its mate M,lyn L,+ A = D,pan dot of arc AB. DK = delta, pan daw of kok BKA,or alpha beta ; Q.E. F. Ek = P" = 2". Wooy P"K = P"G. Neh; pan daw gih kok Ek = Hjelmslev /, + ℓ,lyn U,/Dal- beth & lyn K, = Velsmlejh /, + E = G, way PK = PG & KG = delta,+ Λ = D. MG,ching E, + y = W chung gih yuan psi maw E mwa G; kon booy fo lwen.

A D# joy tzu D gai fat (30 peen siang) = $\sqrt{65 - 7}/2$ = len 0.5311 3 meh. Gai way so P"A = $\sqrt{(20/13)}$ hur, AG so = 2 −$\sqrt{20/13}$. Hur P"A len 1.24 & AG len 0.76 meh. WAN tet BKA? DAH. Alpha = 2x −AY = 2A & Beta = 2x −By=2B hur if S be sia of KA & S' of beta; fat (S' −S)/(1 + S'S) shik 2(A−B)/(4 + AB) or gih ye ping hee of alpha are a, b for x & y & for beta hee ju are a',b', hur a=2 b = −A;a'= 2,b' = −B & fat (ab' − a'b)/(aa' + b'b) tzu (−2B +2A)/(4+AB); or since A=3 & B = −1; tet BKA = 8 meh. The follow- ing from TAI SHU FOOK TSIU (992)is worth repeating.

SUK YAI LWEE GIH JURN. Ming yih shuo chon sang & yih ai boo kao swan yen. Kwoo sung lay tao lo & fong way khwi khu fer tzu ho look koong. (Nim:"round the mount- ain""drivin' six white horses" etc. is the folk version.) Lengjur ping harn chun pien sheng, haw & moong. Ko shiong tung yun dzi & kaw yong kih, sao,ink fat,mo ju lun − shee hwai, shuan & shut. Ah tee zih keng & way moo sheng.Kiong chay way tiong hiong zhang tung kuang kum zer. Siang ju har nai dah gih ah dan yot keen . Tih shih! (Vide the Interlude explanation of nursery rimes,in BOOK IV,Part II by .'.PERDURABO &.'.VIRAKAM according to the Holy Qabalah To each his own! II,40: Tee sikh yen tai lai tih & sikh tee yen kon fun gih AH Thien & Dee. Chay chun!

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